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ABSTRACT

Five segments of the Self-Paced Physics Course materials are presented in this problems and solutions book for use as the first part of student course work. The subject-matter topics are related to significant digits, vectors, dimensions, dot and cross products, speeds, velocities, problem solving procedures, projectiles, Newton's laws, and friction. Contained in each segment are information panels, core problems enclosed in a box, core-primed questions, scrambled problems solutions, and true-false questions. A study guide is provided for each segment; and used to answer the true-false questions and to reveal directions for reaching solutions when the correct answer is chosen. When the core problem is answered incorrectly, the study requires the student to follow the remedial or enabling loop, leading, in turn, to the solution of core-primed questions. Also included is an answer sheet for the five segments. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

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NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY, LONG ISLAND, NEW YORK 11568

Course Materials - Fall, 1970

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ED 075238

UNITED STATES NAVAL ACADEMY

Self-Paced

PHYSICS

SEGMENTS 1-5

SE 016 067

DEVELOPED AND PRODUCED UNDER THE
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PROJECT #8-0446, FOR THE U.S. NAVAL ACADEMY
AT ANNAPOLIS. CONTRACT #N00600-68C-0749.

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY,

PROBLEMS -- AND THEIR CORRECT-ANSWER PAGES

Segments 1, 2, 3, 4 and 5

Segment 1		Segment 2		Segment 3		Segment 4		Segment 5	
P. #	Solution Page	P. #	Solution Page	P. #	Solution Page	P. #	Solution Page	P. #	Solution Page
1	24a	1	33b	1	11a	1	24b	1	18a
2	14a	2	22a	2	17a	2	22a	2	14a
3	19b	3	27c	3	14a	3	23a	3	15b
4	21b	4	19a	4	21a	4	35c	4	21a
5	17a	5	24a	5	12a	5	29a	5	20a
6	21a	6	31a	6	9a	6	25b	6	13a
7	22b	7	21b	7	17b	7	24a	7	17b
8	18a	8	26a	8	15a	8	22b	8	22a
9	15a	9	33a	9	10a	9	26b	9	23a
10	23a	10	22b	10	16b	10	32a	10	19a
11	17b	11	27b	11	21b	11	26a	11	16a
12	20a	12	30a	12	19a	12	22c	12	15a
13	22a	13	25a	13	14b	13	21a	13	24a
14	19a	14	21a	14	24a	14	34a	14	21b
15	22c	15	30b	15	12b	15	27b	15	13c
16	16a	16	27a	16	23a	16	31a	16	17a
		17	28a	17	16a	17	25a	17	25a
		18	23a	18	20a	18	27a	18	25b
eading: *AB Appendix A		19	26b	19	18a	19	29b	19	13b
		20	20a	20	13a	20	38b	20	24b
*HR 1-1/1-6; 2-1/2-3		21	32a			21	20a	21	18b
						22	35b		
		HR*2-4,2-5 *3-1/3-6;4-6		HR*3-7/3-10; *4-1/4-3		23	35a		
						24	38a	*HR 6-1;ch.5(review)	
*SW Intro; 1-1, 1-2;2-3 3-1		SZ 4-1/4-4;4-5, 4-6; *4-9		SZ 4-7,4-8; 6-1/6-5		25	33a		
						26	36b	SZ 6-7;6-8	
		SW 3-1/3-5				27	31b		
						28	30a	AB 25-1/25-5	
		AB 8-1/8-3;8-5				29	36a		
						30	28a		
						31	39a		
						32	37a		
						*HR 5-1/5-9;5-10			
						SZ 2-1/2-5;5-1/5-3			
						AB Chapter 24			

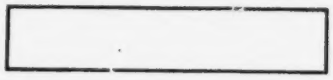
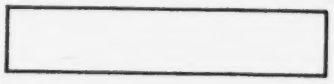
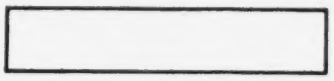
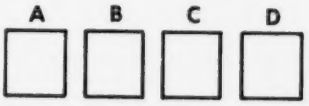
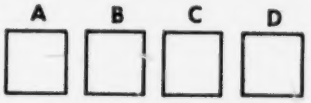
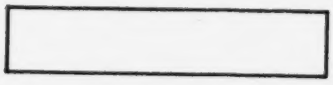
P	STEP	NAME
	0.1	Reading: *AB Appendix A *HR 1-1/1-6; 2-1/2-3 *SW Introduction; 1-1, 1-2; 2-3; 3-1
	0.2	Information Panel, "Significant Digits"
1		A B C D T F
	1.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
2		A B C D
3		_____ (ans)
4		_____ (ans)
5		T F T F _____ (ans)
	5.1	Information Panel, "Vectorial Addition"
6		A B C D T F
	6.1	If your first choice was correct, advance to 9.1; if not, continue sequence.
7		A B C D

P	STEP	SECTION SEGMENT 1
8		A B C D
9		A B C D T F
	9.1	Information Panel, "Resolution of Vectors"
10		A B C D T F
	10.1	If your first choice was correct, advance to 12.1; if not, continue sequence.
11		_____ D_x = _____ (ans) _____ D_y = _____ (ans)
12		_____ R_x = _____ (ans) _____ T F R_y = _____ (ans)
	12.1	Information Panel, "Dimensional Checking"
13		A B C D T F
	13.1	If your first choice was correct, advance to 16.1; if not, continue sequence.

P	STEP	NAME	P	STEP	SECTION	SEGMENT 1
14		A ☐ B ☐ C ☐ D ☐				
15		A ☐ B ☐ C ☐ D ☐				
16		A ☐ B ☐ C ☐ D ☐ T ☐ F ☐				
	16.1	Homework: HR 2-13				

P	STEP	NAME
	0.1	Reading: HR *2-4, 2-5; *3-1/3-6; 4-6 SZ 4-1/4-4; 4-5, 4-6; *4-9 SW 3-1/3-5 AB 8-1/8-3; 8-5
	0.2	Information Panel, "Dot (Scalar) Product"
1		A B C D T F
	1.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
2		_____ (ans)
3		_____ (ans)
4		_____ (ans)
5		A B C D T F
	5.1	Information Panel, "Cross (Vector) Product"
6		T F _____ (ans)
	6.1	If correct, advance to 9.1; if not, continue sequence.

P	STEP	SECTION SEGMENT 2
7		A B C D
8		A B C D
9		A B C D T F
	9.1	Information Panel, "Average Speed and Average Velocity"
10		A B C D T F
	10.1	If your first choice was correct, advance to 13.1; if not, continue sequence.
11		_____ (ans)
12		_____ (ans)
13		A B C D T F
	13.1	Information Panel, "Instantaneous Velocity"
14		T F _____ (ans)
	14.1	If correct, advance to 16.1; if not, continue sequence.
15		A B C D

P	STEP	NAME	P	STEP	SECTION	SEGMENT 2
16						
		α = _____ (ans)				
		 T F				
		β = _____ (ans)				
	16.1	Information Panel, "Relative Velocity"				
17		 T F				
		_____ (ans)				
	17.1	If correct, advance to 21.1; if not, continue sequence.				
18						
		_____ (ans)				
19		 A B C D				
20		 A B C D				
21		 T F				
		_____ (ans)				
	21.1	Homework: HR 4-24				

P	STEP	NAME
	0.1	Reading: HR *3-7/3-10; *4-1/4-3 SZ 4-7, 4-8; 6-1/6-5
	0.2	Information Panel, "Problem Solving Procedures"
1		T F (ans)
	1.1	If correct, advance to P 6; if not, continue sequence.
2		(ans)
3		(ans)
4		(ans)
5		T F (ans)
6		T F (ans)
	6.1	If correct, advance to P 9; if not, continue sequence.
7		(ans)

P	STEP	SECTION SEGMENT 3
8		T F (ans)
9		T F (ans)
	9.1	If correct, advance to 11.1; if not, continue sequence.
10		(ans)
11		T F (ans)
	11.1	Audiovisual, PROJECTILE MOTION
	11.2	Information Panel, "Equation Summary for Projectile Motion"
12		A B C D T F
	12.1	If your first choice was correct, advance to P 18; if not, continue sequence.
13		A B C D
14		A B C D
15		A B C D

P	STEP	NAME	P	STEP	SECTION SEGMENT 3
16		A B C D			
17		T F			
		_____ (ans)			
18		T F			
		_____ (ans)			
	18.1	If correct, advance to 20.1; if not, continue sequence.			
19					
		_____ (ans)			
20		T F T F			
		_____ (ans)			
	20.1	Homework: HR 4-10			

P	STEP	NAME
	0.1	Reading: *HR 5-1/5-9; 5-10 SZ 2-1/2-5; 5-1/5-3 AB Chapter 24
	0.2	Information Panel, "Newton's First Law of Motion Applied"
	0.3	Audiovisual, NEWTON'S FIRST LAW
1		A B C D T F
2		T F _____ (ans)
	2.1	If correct, advance to P 5; if not, continue sequence.
3		A B C D
4		T F _____ (ans)
5		A B C D T F
	5.1	Information Panel, "Newton's Second Law of Motion Applied"
	5.2	Audiovisual, NEWTON'S SECOND LAW
6		A B C D T F
	6.1	If your first choice was correct, advance to P 11; if not, continue sequence.
7		_____ (ans)
8		A B C D

P	STEP	SECTION — SEGMENT 4
9		A B C D
10		A B C D T F
11		T F _____ (ans)
	11.1	If your first choice was correct, advance to 15.1; if not, continue sequence.
12		A B C D
13		A B C D
14		A B C D
15		T F _____ (ans)
	15.1	Information Panel, "Newton's Third Law of Motion Applied"
	15.2	Audiovisual, NEWTON'S THIRD LAW
16		A B C D T F
	16.1	If your first choice was correct, advance to P 21; if not, continue sequence.
17		A B C D
18		A B C D

P	STEP	NAME
19		A B C D
20		A B C D T F
21		T F (ans)
	21.1	If correct, advance to P 26; if not, continue sequence.
22		A B C D
23		(ans)
24		A B C D T F
25		(ans)
26		T F (ans)
	26.1	If correct, advance to P 29; if not, continue sequence.
27		A B C D
28		T F (ans)
29		A B C D T F

P	STEP	SECTION SEGMENT 4
	29.1	If correct, advance to P 32; if not, continue sequence.
30		A B C D
31		A B C D T F
32		a_x = (ans)
		$\vec{R}$ = (ans)
		T F N = (ans)
	32.1	Homework: HR Problem 5-6

P	STEP	NAME
	0.1	Reading: *HR 6-1; ch. 5 (review) SZ 6-7; 6-8 AB 25-1/25-5
	0.2	Information Panel, "Newton's Laws of Motion Combined"
1		A B C D T F
2		T F (ans)
	2.1	If correct, advance to 4.1; if not, continue sequence.
3		A B C D
4		T F (ans)
	4.1	Information Panel, "Coefficients of Friction"
5		A B C T F
	5.1	If your first choice was correct, advance to 9.1; if not, continue sequence.
6		A B C D
7		T F (ans)
8		(ans)

P	STEP	SECTION SEGMENT 5
9		A B C T F
	9.1	Information Panel, "Friction on an Inclined Plane"
10		T F (ans)
11		T F (ans)
12		T F T F (ans)
	12.1	Audiovisual, ATWOOD'S MACHINE
13		A B C D T F
	13.1	If your first choice was correct, advance to P 18; if not, continue sequence.
14		A B C D
15		A B C D
16		A B C D
17		A B C D T F
18		T F (ans)

P	STEP	NAME	P	STEP	SECTION SEGMENT 5
	18.1	If correct, advance to 21.1; if not, continue sequence.			
19		[ ] (ans) _____			
20		[ ] (ans) _____			
21		[ ] T [ ] F (ans) _____			
	21.1	Homework: HR Problem 6-8			

INFORMATION PANEL Significant Digits

OBJECTIVE

To interpret and apply the rules for significant digits (or figures) to arithmetical operations as they appear in physics problems.

A substantial portion of the reading for this Segment is devoted to the measurement of length, time, and mass. Although this material is not particularly emphasized in the PROBLEM AND SOLUTIONS, you are urged to give the reading as much time as it requires for thorough understanding. Much of it may be familiar to you, some of it may not; but, in any event, questions dealing with units of measurement will appear in subsequent Progress Checks and Diagnostic Tests.

Most students starting a course in college physics have not had much exposure to the idea of significant digits in expressing physical quantities. The system of significant digits we shall use is primarily designed for use in the laboratory and is intended to provide some indication of the precision with which measurements are made. If you measure the diameter of a U.S. quarter with an ordinary centimeter scale, you would be justified in writing it as 2.39 cm because the scale permits you to read the first two digits without guessing and you can estimate the third digit. With a vernier caliper, you might write the diamter as 2.387 cm, in this case expressing the result to 4 significant digits. A micrometer caliper would enable you to record the diameter as 2.3869 cm (5 significant digits). Each successive measurement was more precise because the instrument itself is capable of improved precision because of better design and construction.

In order to apply the rules for handling significant digits in arithmetic operations, you must develop the ability to recognize the number of significant digits in a laboratory measurement or in problem data. A few simple rules with examples are given below:

(1) *The last digit in the number is always uncertain.* A sample twenty-dollar bill measured with a student scale yields a width measurement of 6.63 cm. The last digit is obtained by estimating where the edge of the bill falls between marked divisions. The scale informs the observer with certainty that the width is between 6.6 and 6.7 cm. He estimates the position of the edge to be about one-third of the way from the 6.6 cm mark to the 6.7 cm mark.

(2) *Any digit other than zero is always significant.* Thus, 5.77 contains 3 significant digits, 8.742 contains 4 significant digits, and 2.997925 contains 7 significant digits.

next page

continued

(3) *A zero is significant if any digit* other than zero *precedes it in the number, otherwise it is not.* (An exception to this rule will be discussed shortly.) Some examples of rule (3) are given below:

5.08--the zero is significant; 3 significant digits;

8.006--both zeros are significant; 4 significant digits;

0.042--neither zero is significant. The first zero is used merely to emphasize the presence of the decimal point while the second zero locates the position of the decimal point. This number contains only 2 significant digits;

0.0706--the first two zeros are not significant but the third zero is significant since it is preceded by the digit "7." This number therefore contains 3 significant digits;

3.8000--all three zeros are significant; this number contains 5 significant digits.

(4) Exception to Rule (3): *If a number contains no decimal point, and if the number terminates in one or more zeros, the zeros may or may not be significant.* An example will be most helpful for this rule. Many elementary textbooks give the distance between the moon and the Earth as 240,000 miles. How many zeros are significant? How many significant digits does this number have? Expressed in this form, there is insufficient information given to provide the answers. A reliable atlas states that the mean distance to the moon is 238,856 miles--a measurement with 6 significant digits. Obviously, this number has been rounded off to yield 240,000 miles in the first expression of distance so that even the "4" is uncertain, hence there are only 2 significant digits in it. Some authorities terminate such numbers with a decimal to specify the significant digit count, but there is a much better way to do it than this. By using exponential notation (powers of 10), all doubt can be immediately removed. The following examples show this clearly:

240,000 expressed as 2.4×10^5 specifies 2 significant digits;

93,000,000 expressed as 9.30×10^7 specifies 3 significant digits;

37,000 expressed as 3.70×10^4 specifies 3 significant digits, but the same number written as 3.7×10^4 specifies only 2 significant digits;

238,857 expressed as 2.38857×10^5 specifies 6 significant digits.

next page

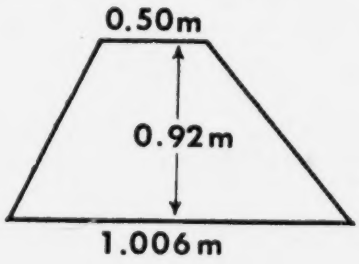
continued

The problems in this section require that you be able to

(a) determine by inspection the number of significant digits in a given number;

(b) apply the rules for significant digits in multiplication, division, addition, etc.

1.



The dimensions of the trapezoid shown in the figure were measured with different instruments. The area of the trapezoid should be written as:

A. 0.69276 m^2
B. 0.69 m^2
C. 0.6946 m^2
D. 0.7 m^2

2. Choose the group in which all the numbers involved have three significant figures.

A. 00.3; 31.4; 667.0
B. 0.300; 714; 0.0410
C. 0.30; 0.500; 60034
D. 1.30; 0.50; 714

3. How many significant figures should be present in the sum: 15.811 + 0.6552 + 74.3 + 10.006?

4. What is the volume of a sphere having a radius of 3.00 cm? (take π as 3.1416)

5. True or false? The product 1.3(0.51 + 1.2001) should be written with *two* significant digits.

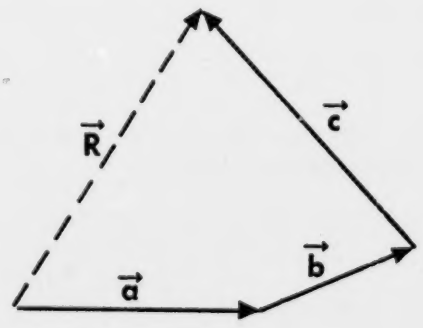
INFORMATION PANEL — Vectorial Addition

OBJECTIVE

To use either the graphical (geometric) or analytical method in the process of obtaining the resultant of two or more vectors.

A vector quantity such as displacement or velocity always has associated with it (a) magnitude and (b) direction. When two or more vector quantities are to be added to determine the magnitude and direction of the resultant (vector sum), the addition process must be performed vectorially rather than algebraically.

Graphical Method: This is a geometric method in which the vectors are drawn head-to-tail at the prescribed angle to each other. If the vector quantities are such as to yield a resultant of zero, the arrows will form a closed polygon. When the resultant is not zero, its magnitude and direction is given by the arrow needed to close the polygon. See vector $\vec{R}$ in the figure at right. The polygon method may be used to find the resultant of any number of vector quantities if the magnitudes and directions of all of them are known. Vectors may be summed in any order (commutative law) or in any grouping (associative law).



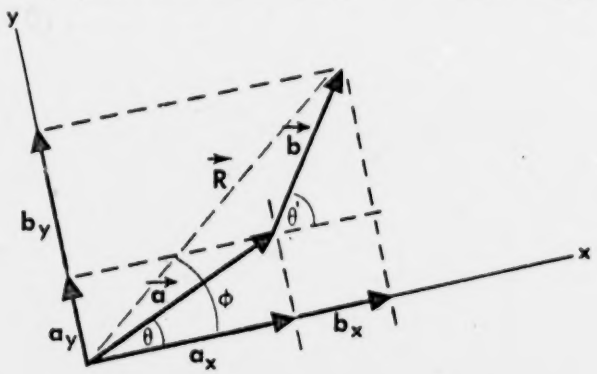
Graphical Method of Vectorial Addition

One vector may be subtracted from another by recognizing that $\vec{a} - \vec{b}$ may be written as $\vec{a} + (-\vec{b})$. Thus, in subtracting $\vec{b}$ from $\vec{a}$, we reverse the direction of $\vec{b}$ to make it $(-\vec{b})$ and then add it to $\vec{a}$ by the process previously described.

next page

continued

Analytical Method: Any given vector can be replaced by a set of components at right angles to one another called *rectangular components;* such components are projections of the vector arrow on the x- and y-coordinate axes. Since these axes may be selected in any orientation with respect to the original vector, the rectangular components obtained for one set of axes will differ from those for another set. Thus, a given pair of components are not unique in either magnitude or direction.



Analytical Method of Vectorial Addition

As indicated in the accompanying diagram, if vector $\vec{a}$ makes an angle θ with the x-axis, then the x-component is given by $a_x = a\cos\theta$ where a is the magnitude of the original vector, and the y-component by $a_y = a\sin\theta$. To add a number of vectors, you would find the net value of all the x-components by algebraic addition, the net value of all the y-components in the same way, and the resultant by using the Pythagorean theorem:

$$|\vec{R}| = \sqrt{(a_x + b_x)^2 + (a_y + b_y)^2}$$

The direction of the resultant found by the analytical method is obtained from the relationship:

$$\tan\phi = \frac{a_y + b_y}{a_x + b_x}$$ where ϕ is the angle between the resultant and the x-axis

The analytical process may be extended in a similar manner to any number of initial vectors.

The choice of coordinate axes for a given summation is up to you. If these are selected judiciously, your work can be greatly simplified. The problems in this section demonstrate this point clearly. To solve them you will be expected to be able to

(a) use the graphical method for addition of vectors;
(b) use the analytical method for addition of vectors.

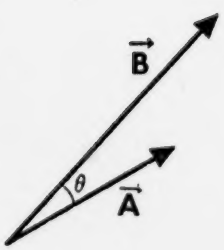
6. A plane travels 40 miles due north, then changes its course to a direction of 37° east of north and travels for 50 miles. Finally it travels for 30 miles due east. Its total displacement is:

A. 100 miles
B. 120 miles
C. 100 miles at 37° east of north
D. 120 miles at 37° east of north

7. Which of the following is *not* a vector quantity?

A. displacement
B. time
C. velocity
D. force

8.



Two vectors $\vec{A}$ and $\vec{B}$ make an angle of θ with one another as shown in the figure. The magnitude of the resultant of $\vec{A}$ and $\vec{B}$ is

A. $R^2 = A^2 + B^2$

B. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

C. $R = A + B$

D. $R = \sqrt{A^2 + B^2 + AB\tan\theta}$

9. A car travels 40 miles due southwest, then changes its course to due east for 50 miles. Finally it travels for 80 miles in a direction 37° north of east. What is its total displacement?

A. 85 miles
B. 88 miles
C. 19.7 miles, 13° east of north
D. 88 miles, 13° north of east

INFORMATION PANEL Resolution of Vectors

OBJECTIVE

To solve problems in which vector quantities must be resolved into rectangular components.

The process of vector resolution is the reverse of that of vector addition: you are required to find a pair of rectangular components which, when added vectorially, will yield the correct direction and magnitude of the vector which you initially resolved. As we have already pointed out, the nature of the components you obtain in the resolution process depends upon the orientation of the axes you select. Careful choice of axes can do much to simplify many problems as the following example illustrates.

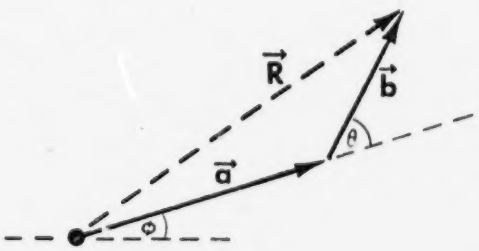


Figure 1

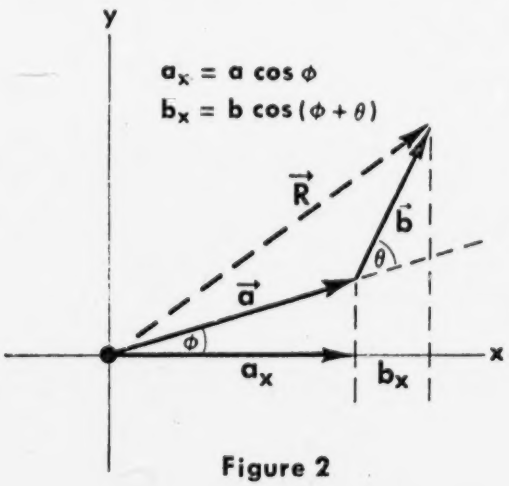


Figure 2

Suppose you were asked to find the resultant of $\vec{a}$ and $\vec{b}$ in the diagram of Figure 1. This means that you are to find the magnitude of $\vec{R}$ and the angle it makes with, say, vector $\vec{a}$. To use the analytical method, you would want to start by resolving $\vec{a}$ and $\vec{b}$ into their x-components so it would be necessary to select the x-axis (and, of course, the y-axis at the same time).

It would be quite natural to choose the axes as illustrated in Figure 2, but if you do that, the expressions for the components a_x and b_x come out unnecessarily complicated. The same would be true of the y-components when these axes are used. (Try it.)

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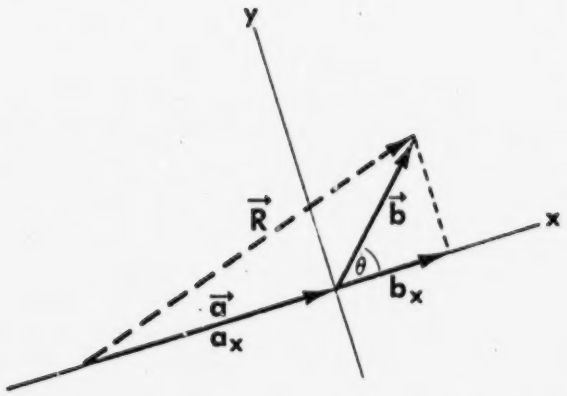


$$a_x = a$$

$$b_x = b \cos\theta$$

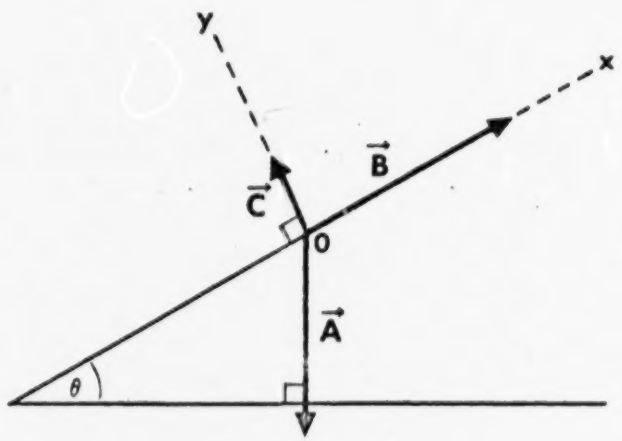
Figure 3

If the axes are drawn as shown in Figure 3, however, with the x-axis coinciding with a_x, the required relationships become simpler for both the x- and y-components. Thus, before you start working on problems of this and similar varieties, study the requirements before establishing the orientation of the axes; it will be worth your while in time and effort saved.

In this portion of your work you will be required to solve a series of problems in which rectangular resolution is a key operation.

next page

10.



Find the components R_x and R_y of the vector $\vec{R}$, where vector $\vec{R}$ is the resultant (sum) of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. Use the coordinate system indicated.

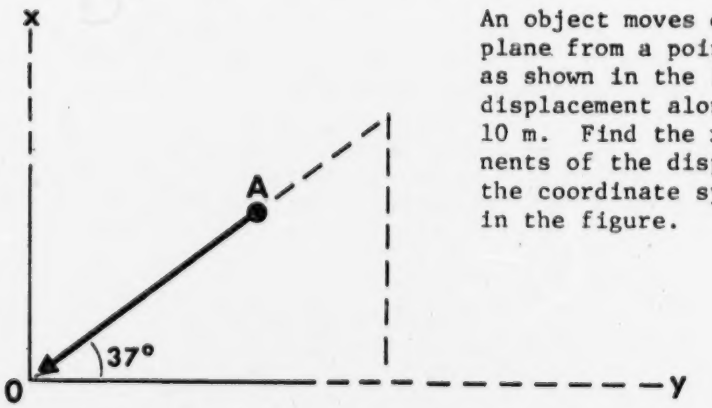
A. $R_x = A \sin\theta$ $R_y = A \cos\theta$

B. $R_x = B \cos\theta$ $R_y = A \sin\theta$

C. $R_x = B - A \sin\theta$ $R_y = C - A \cos\theta$

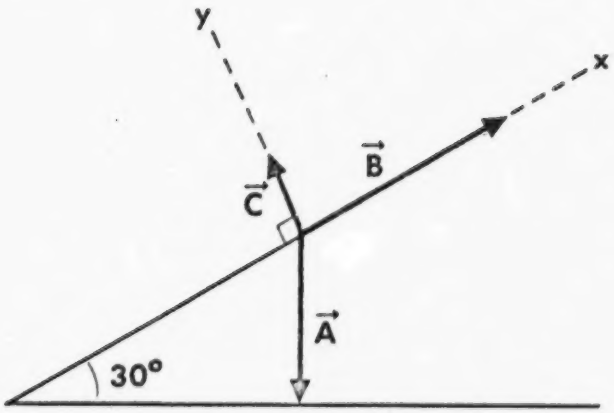
D. $R_x = C - B \cos\theta$ $R_y = A - B \sin\theta$

11.



An object moves down an inclined plane from a point A to point O, as shown in the figure. Its displacement along the plane is 10 m. Find the x and y components of the displacement in the coordinate system as indicated in the figure.

12.



Find the components R_x and R_y of the vector $\vec{R}$ when vector $\vec{R}$ is the resultant of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ whose magnitudes are 6, 8, and 2 units, respectively. Use the coordinate system indicated.

INFORMATION PANEL

Dimensional Checking

OBJECTIVE

To practice unit substitutions in symbolic equations as a means of dimensional checking.

Symbolic equations in physics almost invariably represent real situations involving actual objects in the physical environment. The characteristics of these objects and events are generally measurable in terms of fundamental or derived units. Hence, when we write, for the area A of a circle of radius r,

$$A = \pi r^2$$

we are saying that the left side of the equation (term A) is the *exact equivalent of the term on the right side of the equation* (πr^2) in all respects--nature, units of measure, magnitude, etc.

A simple method exists for partially checking the validity of a newly derived or converted equation. We merely substitute the correct units for all the quantities in both sides to be certain that this results in an identity.

next page

continued

For example, the interest paid by a savings bank on a deposit before compounding is calculated with the help of the equation

$$I = PRt$$

where I = interest in dollars
P = principal in dollars
R = interest rate in percent per annum
t = time of accrual in years (or fraction of years)

To check the validity of the equation, we substitute units as follows:

$$\underset{\text{(dollars)}}{I} = \underset{\text{(dollars)}}{P} \times \underset{\left(\frac{\%}{\text{yr}}\right)}{R} \times \underset{\text{(yr)}}{t}$$

Percentage is a pure number, that is, it is dimensionless and has no units of measure. Replacing the percentage symbol by "1" to keep the fraction, we then write

$$\text{dollars} = \text{dollars} \times \frac{1}{\text{yr}} \times \text{yr}$$

Units may be handled like numbers; so in the statement above, the "yr" terms may be cancelled out leaving

$$\text{dollars} = \text{dollars}$$

--a dimensional identity.

Consider another simple example. The average speed of a train can be determined by dividing the total distance the train travels by the time required for the trip, or

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time for trip}}$$

We know that average speed is measured in miles per hour which is normally written as mi/hr in which the fraction bar is always read as "per." Thus, the right side of the equation must come out in miles per hour, too, if we are to end up with an identity. Substituting units on both sides,

$$\frac{\text{mi}}{\text{hr}} = \frac{\text{mi}}{\text{hr}}$$

which demonstrates that the equation is at least dimensionally correct.

next page

continued

You may have noticed that the dimensional method offers a *partial* check on the validity of an equation. The reason for this limitation is easily seen by referring back to our first equation, the area of a circle = $A = \pi r^2$. A dimensional check shows that

$$\underset{\text{(square ft)}}{\text{area}} = \underset{\text{(dimensionless)}}{\pi} \times \underset{\text{(square ft)}}{r^2}$$

$$ft^2 = ft^2$$

But note that this check would have worked out the same way if the constant had been π^2 or π^3 or $\sqrt{\pi}$. So, you can see that dimensional checking cannot reveal an error in a dimensionless constant. You will find, however, that this shortcoming is not serious and that you will make frequent use of the system for verifying your work.

The problems in this section are very simple, indeed. They are intended to provide some practice with dimensional checking in a preliminary way. Although the equations you meet as your work proceeds will be more complicated (as indeed they should!), exactly the same principles are utilized for checking them.

13. A car moving at a constant rate R covers a distance D during a time interval T. Its rate can be expressed in

A. ft-min
B. sec per ft
C. yd per hr
D. mi-hr

14. The three sides of a given triangle are, respectively, 5 in, 12 in, and 13 in long. In which unit should the area be expressed?

A. in^2
B. in
C. in^3
D. $\sqrt{in^3}$

15. An equation relating to a right cylinder is given as

$$x = \pi r^2 L$$

If r is the cross-sectional radius in square inches and L is its axial length, then x should come out in

A. in^2
B. in^3
C. in^4
D. $\sqrt{in}$

16. The distance traveled by a car starting from rest and picking up speed (accelerating) uniformly is symbolically expressed by the equation

$$d = \frac{1}{2} at^2$$

where d = distance in meters and t = time in seconds. The dimension of the acceleration a would then be expressed in

A. meter-seconds squared
B. meters per second
C. meters squared per second
D. meters per second squared

[a] CORRECT ANSWER: B

B - The zero before the decimal point in the number 0.300 is not significant. It is merely a convention used to show that a decimal point follows. The zeros following the 3, however, are only placed there if they are significant. Hence, 3 significant figures. There can be no question, of course, about the number 714. Finally, in the last number 0.0410, the first two zeros, those on either side of the decimal point, are not significant while the zero at the end is significant. The second zero is used merely to locate the decimal. Again, 3 significant figures.

A - INCORRECT - Two of the three numbers given do not fit the category of three significant figures.

00.3 is improperly written since the first zero has no meaning whatever. This number should be written as 0.3. The zero in this case is not significant; it is inserted merely to show that a decimal point does exist before the "3." Thus, 0.3 has but one significant figure.

667.0 has 4, not 3, significant figures. The zero after the decimal point *is* significant. It shows that the number is closer to 667.0 than it is to 666.9 or to 667.1.

C - INCORRECT - In this group there is only one number having 3 significant figures: 0.500. The zero before the 5 is *not* significant. It merely shows that a decimal point is present right after it. Both of the zeros after the 5 *are* significant.

Similarly, in 0.30 there are only two significant figures; namely, the 3 and the zero to its right.

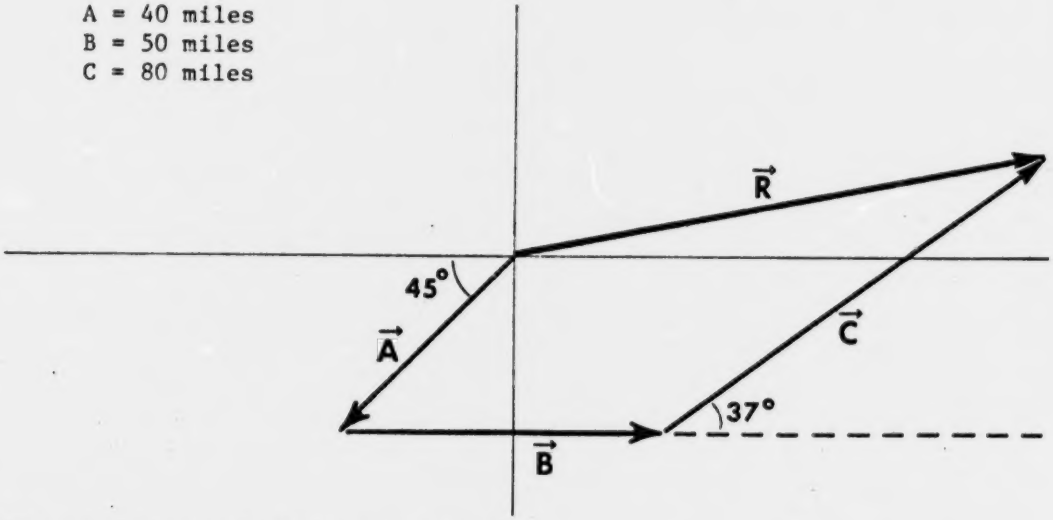
In the number 60034, the two zeros are just as significant as any other digits in these positions would be. Hence, 60034 has five significant figures.

D - INCORRECT - The first and third numbers have 3 significant figures. The number 0.50, however, has only 2 significant figures; namely, the 5 and the zero to its right. The zero to the left of the 5 is not significant. It is merely used to show that a decimal point follows.

[a] CORRECT ANSWER: D

Figure 1 below shows the three displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ representing the three parts of the trip. These displacements must be added vectorially. The magnitudes of these vectors are given as

A = 40 miles
B = 50 miles
C = 80 miles



The components of these vectors are

x-component	y-component
$A_x = -40 \cos 45° = -28.28$	$A_y = -40 \cos 45° = -28.28$
$B_x = 50$	$B_y = 0$
$C_x = 80 \cos 37° = 64$	$C_y = 80 \sin 37° = 48$

The total displacement R is given by the sum of three vectors:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

The components R_x and R_y are

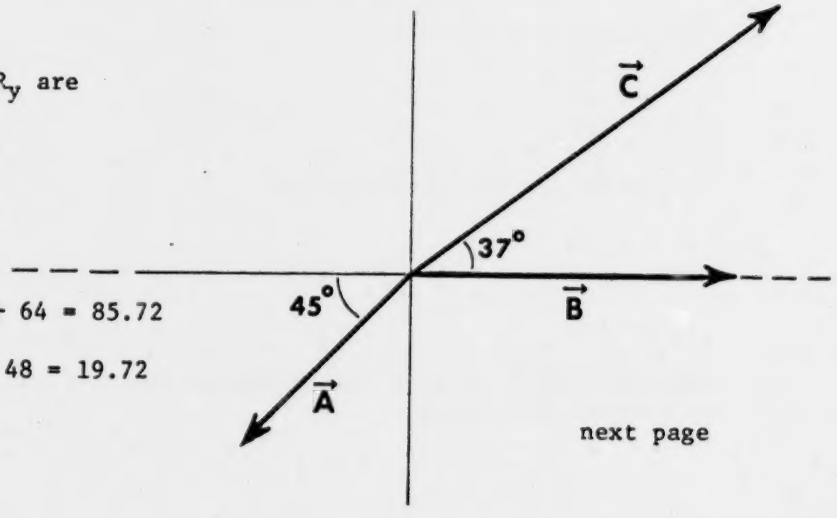
$$R_x = A_x + B_x + C_x$$

$$R_y = A_y + B_y + C_y$$

Substituting:

$$R_x = -28.28 + 50 + 64 = 85.72$$

$$R_y = -28.28 + 0 + 48 = 19.72$$



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The magnitude of the total displacement $\vec{R}$ is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(85.72)^2 + (19.72)^2} = 88 \text{ miles}$$

and the direction of $\vec{R}$ is given by

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{19.72}{85.72}\right) = \tan^{-1}(0.23) = 13°$$

where θ is to be measured from the east in the counterclockwise direction.

Thus, the total displacement $\vec{R}$ is 88 miles at 13° north of east.

TRUE OR FALSE? If the magnitudes of $\vec{A}$, $\vec{B}$, and $\vec{C}$ had all been 40 miles, the resultant $\vec{R}$ would have fallen in the 4th quadrant.

[a] CORRECT ANSWER: D

To check dimensionally, first solve the symbolic equation for the unknown a:

$$d = \frac{1}{2} at^2$$

so

$$a = \frac{2d}{t^2}$$

Substitute the given units

$$a = \frac{(2)(m)}{(sec)^2}$$

and since (2) is dimensionless

$$a = \frac{m}{sec^2} \text{ or meters per second squared}$$

TRUE OR FALSE? Another satisfactory way of reading $a = m/sec^2$ is "meters per second per second."

[a] CORRECT ANSWER: TRUE

The product = 1.3(0.51 + 1.2001)
= 1.3(1.7101)

However, the number of significant digits in a sum is determined by letting the sum have the same number of *decimal places* as the number with the fewest decimal places. In this example 0.51 is that number which has fewer decimal places of the two numbers 0.51 and 1.2001. Hence, the sum of 0.51 and 1.2001 is 1.71.

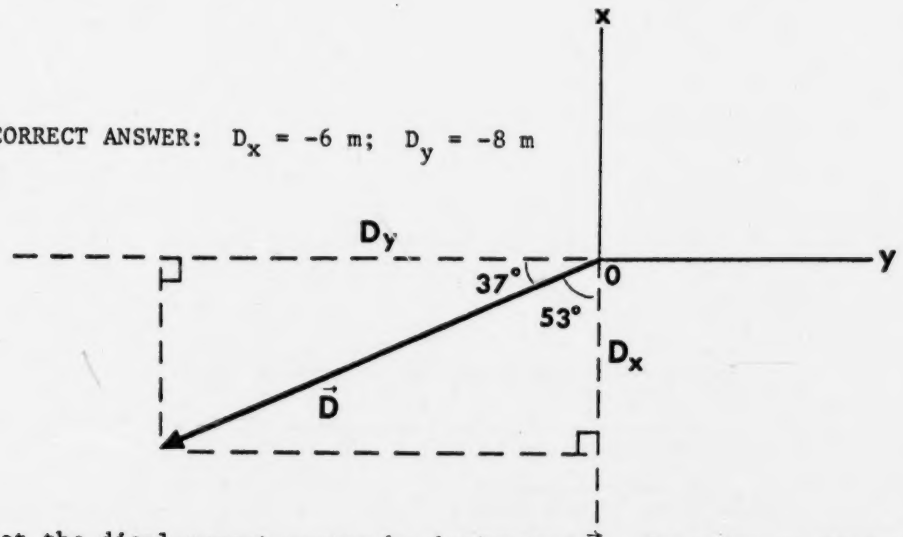
Therefore,

$$\text{the product} = 1.3 \times 1.71 = 2.223$$

For a product of two or more numbers, the answer should have the same number of significant digits as the component number with the fewest significant digits. Hence the product is written as *2.2* with 2 significant digits.

TRUE OR FALSE? In the above solution, the number 1.7101 has 4 significant digits.

[b] CORRECT ANSWER: $D_x = -6$ m; $D_y = -8$ m

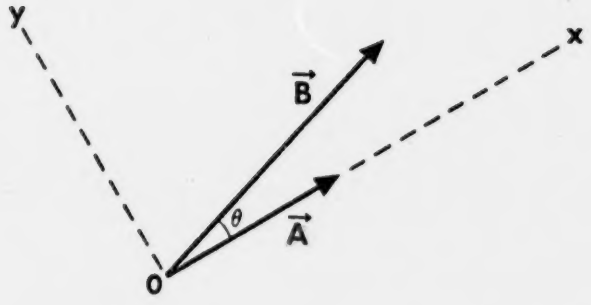


Let the displacement vector be designated $\vec{D}$. In order to find the components of a vector in a given coordinate system, one draws the vector with its tail starting at the origin of the coordinate system. From the figure one finds

$$D_x = -D \cos 53° = -10 \text{ m} \times 0.6 = -6 \text{ m}$$

$$D_y = -D \cos 37° = -10 \text{ m} \times 0.8 = -8 \text{ m}$$

[a] CORRECT ANSWER: B



Let us choose a coordinate system such that the x-axis coincides (is parallel) with the direction of vector $\vec{A}$.

$$\vec{R} = R_x \hat{\imath} + R_y \hat{\jmath}$$

The components of the resultant are found using

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

so that

$$R_x = A + B\cos\theta$$

and

$$R_y = B\sin\theta$$

The magnitude of $\vec{R}$ is found from the Pythagorean theorem,

$$R^2 = R_x^2 + R_y^2$$

Hence

$$R = \sqrt{(A^2 + B^2\cos^2\theta + 2AB\cos\theta) + B^2\sin^2\theta}$$

or

$$R = \sqrt{A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

where the identity, $\cos^2\theta + \sin^2\theta = 1$, has been utilized.

Note: The above result is essentially the law of cosines in trigonometry.

[a] CORRECT ANSWER: A

The area of a triangle is found by using the relation $A = \frac{1}{2}$ bh where b = length of base and h = length of altitude.

Substituting:

$$A = \frac{1}{2} \text{ (length unit)(length unit)}$$

Since 1/2 is a dimensionless constant, the final dimension is

$$A = \text{(length)(length)} = \text{(length)}^2$$

hence the answer must be in^2.

[b] CORRECT ANSWER: Four

The number of significant figures in a sum should normally be determined by letting the sum have the same number of *decimal places* as the number with the fewest decimal places. In this example, 74.3 is that number. Hence the sum need not show more than 1 digit after the decimal. Thus,

$$\begin{array}{rr} & 15.811 \\ & 0.6552 \\ & 74.3 \\ & 10.006 \\ \hline \text{SUM} & 100.7722 \longrightarrow 100.8 \end{array}$$

The correct way of writing this sum is 100.8, which has 4 significant figures.

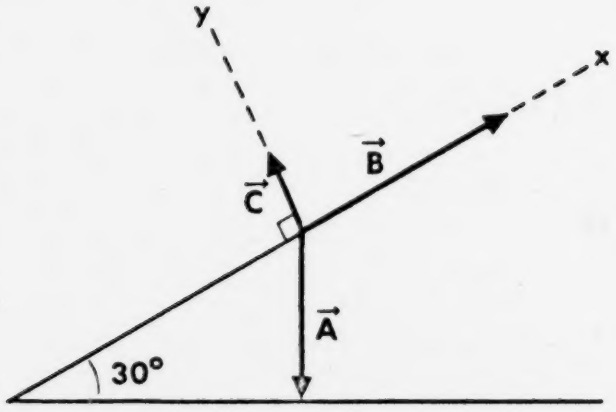
This procedure is quite logical. What we are doing here is adding four numbers which were presumably measured with instruments of varying degrees of precision. The precision of the sum, however, should reflect the least precise measurement. The number 74.3, for some reason or other, is known only to one decimal place; hence, the sum should also be written to one decimal place.

[a] CORRECT ANSWER: $R_x = 5$ units; $R_y = -3.2$ units

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

where $$R_x = A_x + B_x + C_x \qquad (1)$$

$$R_y = A_y + B_y + C_y \qquad (2)$$



The components of the given vectors may be found from the diagram:

$$\left.\begin{aligned} A_x &= -A\cos 60^\circ = -6(1/2) = -3 \\ A_y &= -A\cos 30^\circ = -6(\sqrt{3}/2) = -3\sqrt{3} \\ B_x &= B = 8 \\ B_y &= 0 \\ C_x &= 0 \\ C_y &= C = 2 \end{aligned}\right\} \qquad (3)$$

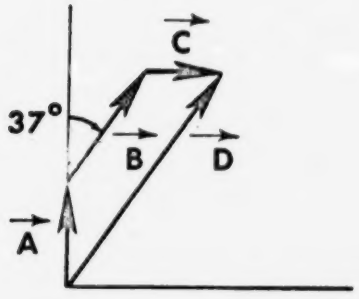
Substituting the values from equation (3) in (1) and (2), we obtain

$$R_x = -3 + 8 + 0 = 5 \text{ units}$$

$$R_y = -3\sqrt{3} + 0 + 2 = -3.2 \text{ units}$$

TRUE OR FALSE? The y-component of $\vec{A}$ is negative because this component is located on the (-y) axis.

[a] CORRECT ANSWER: C



The figure to the left shows the three displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ representing the three legs of the trip. These displacements must be added vectorially. The magnitudes of these vectors were given as A = 40 mi, B = 50 mi, and C = 30 mi. The components of $\vec{A}$ and $\vec{C}$ are:

$$A_x = 0 \qquad A_y = 40 \text{ mi}$$

$$C_x = 30 \text{ mi} \qquad C_y = 0$$

Using the fact that 37° is very nearly the angle of a 3-4-5 right triangle, we find the components of $\vec{B}$; i.e., $B_x = 30$ mi and $B_y = 40$ mi. The total displacement will have components $D_x = A_x + B_x + C_x = 60$ mi, and $D_y = A_y + B_y + C_y = 80$ mi. Again using the 3-4-5 triangle relations, we conclude that

$$\vec{D} = 100 \text{ mi at } 37° \text{ east of north}$$

TRUE OR FALSE? The magnitude of the y-component of vector $\vec{C}$ should be preceded by a (+) sign.

[b] CORRECT ANSWER: 113 cm^3

The volume of a sphere is given by

$$V = (4/3)\pi r^3$$

For r = 3.00 cm the volume becomes

$$V = (4/3)(3.1416)(3.00)^3 \text{ cm}^3$$

$$= 113.04 \text{ cm}^3$$

The number of significant figures should be the same as that in the value of the radius (namely 3), since the constant 4/3 is precisely known and the value of π is given to more than 3 significant figures. Thus the volume of this sphere should be written as $V = 113 \text{ cm}^3$.

[a] CORRECT ANSWER: C

This problem illustrates a very simple kind of dimensional check. The basic relationship between D, R, and T is

$$D = RT \qquad (1)$$

To determine the nature of the unit that must be associated with R, you should first solve equation (1) for R:

$$R = \frac{D}{T}$$

Since distance D is measured in length units (ft, mi, yd, in, etc.) and time T is measured in time units (hr, min, sec, yr, etc.), then R must come out

$$R = \frac{\text{length unit}}{\text{time unit}} \qquad (2)$$

Of the choices given, only answer C checks dimensionally since

$$R = \frac{\text{yd}}{\text{hr}} \quad \text{is read as yd per hr}$$

TRUE OR FALSE? Another acceptable unit for R would have been ft/min.

[b] CORRECT ANSWER: B

A single number (along with the unit system used) is all that is needed to specify elapsed time for an event. Hence, time is a scalar, *not* a vector quantity. The other three quantities--displacement, velocity, and force--are *always* associated with direction and are *always* handled as vector quantities.

[c] CORRECT ANSWER: B

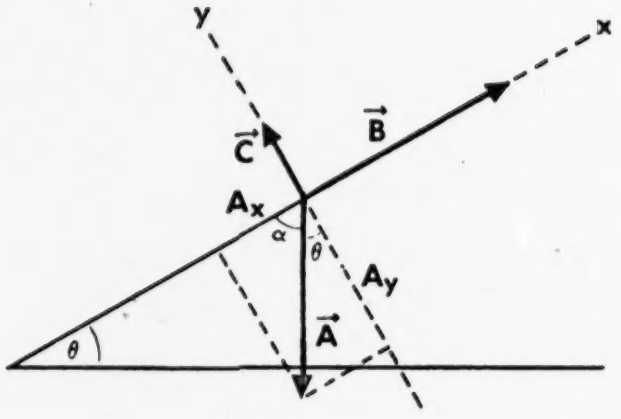
Dimensionally

$$x = \underset{\text{(dimensionless)}}{\pi} \quad \underset{(\text{in})^2}{r^2} \quad \underset{(\text{in})}{L}$$

$$x = (\text{in})^2(\text{in}) = \text{in}^3$$

Thus, x has the dimensions of a volume.

[a] CORRECT ANSWER: C



$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

where

$$R_x = A_x + B_x + C_x \qquad (1)$$

and

$$R_y = A_y + B_y + C_y \qquad (2)$$

The components of the given vectors may be found from the diagram:

$$\left.\begin{array}{ll} A_x = -A\cos\alpha & A_y = -A\cos\theta \\ & (\text{where } \alpha + \theta = 90°) \\ B_x = B & B_y = 0 \\ C_x = 0 & C_y = C \end{array}\right\} \qquad (3)$$

A component of a vector in a given direction is obtained by dropping a perpendicular from the tip of the vector to the given line (i.e., projection).

Substituting the values from equation (3) in (1) and (2), we obtain

$$R_x = -A\cos\alpha + B + 0 = B - A\sin\theta$$

$$R_y = -A\cos\theta + 0 + C = C - A\cos\theta$$

TRUE OR FALSE? The quantity A_x in the above is the horizontal component of $\vec{A}$.

[a] CORRECT ANSWER: B

The area of the trapezoid is given by

$$\text{Area} = \frac{1}{2}(0.92)(0.50 + 1.006)\ \text{m}^2$$

or

$$= \frac{1}{2}(0.92)(1.506)\ \text{m}^2$$

However, the number of significant digits in a sum is determined by letting the sum have the same number of *decimal places* as the number with the fewest decimal places. In this example, 0.50 is that number which has fewer decimal places of the two numbers, 1.006 and 0.50. Hence, the sum of 1.006 and 0.50 is 1.51.

Therefore,

$$\text{Area} = \frac{1}{2}(0.92)(1.51)\ \text{m}^2$$

$$= 0.6946\ \text{m}^2$$

For a product of two or more numbers, the answer should have the same number of significant digits as the component number with the fewest significant digits (i.e., two significant digits), since the constant 1/2 is precisely known.

Therefore,

$$\text{Area} = 0.69\ \text{m}^2$$

TRUE OR FALSE? The number of significant digits in a product of two or more numbers should be the same as the number of significant digits in the least precise component number.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Dot (Scalar) Product

OBJECTIVE

To solve fundamental problems involving the dot or scalar product of two vectors.

Your required reading for this Segment contains an excellently organized section on the subject of multiplication of vectors. It would be presumptuous to attempt to rephrase this section since we feel that the level and treatment are quite right. Perhaps a minor reversal of the emphasis will help you follow this development, however.

Certain operations in mathematics and physics are defined as they are for only one reason: they turn out to be useful in other phases of physics when defined in this particular way. In setting up descriptions of operations, it is frequently possible to define terms and organize a procedure in a variety of ways but it is generally found that one of these patterns works better than the others when applied to other aspects of science. For example, Euclidean and non-Euclidean geometries are equally logical and valid yet seldom does one find the latter type usurping the former in navigation, architecture, or university physics. Why? Simply because Euclidean geometry is more useful in its applications.

You have already had some experience in adding vectors and you are about to investigate the subject of multiplication of vectors. In the real world of objects and events, there are cases where the multiplication of two vectors results in a quantity that has all the characteristics of a scalar; however, there are other cases where a vector multiplied by another vector yields a resultant quantity with all the earmarks of a vector. There must therefore be two different multiplication processes that can be applied to a pair of vectors. The first of these--vector times vector equals scalar--is called the *scalar* product and is symbolized by a dot between the two vectors, hence it is also known as a dot product.

It is important to note at this point that *we are starting out with the predetermination that, whatever method we use to define the operation of obtaining a scalar product, the final definition we adopt must be useful throughout physics.*

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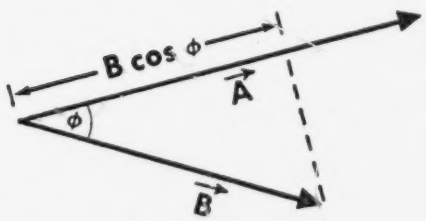
Suppose we find that when we multiply a certain vector $\vec{A}$ by another vector B, we obtain a resultant quantity that is a scalar. We would then start by writing the product as $\vec{A}\cdot\vec{B}$, or in words, "A dot B." We would then look for a reasonable way to define the multiplication operation. Referring to the diagram, if $\vec{A}$ and $\vec{B}$ are oriented to form the angle ϕ between them, and if we define the dot product as the *magnitude of* $\vec{A}$ multiplied by the magnitude of $\vec{B}$ times the cosine of the angle between them, then we can write:



$$\vec{A}\cdot\vec{B} = AB\cos\phi$$

But, as you can see by looking at the diagram, $B\cos\phi$ is merely the projection of $\vec{B}$ on $\vec{A}$, so that what we are really doing is multiplying the magnitude of vector $\vec{A}$ by the magnitude of the projection of vector $\vec{B}$ on the other vector. The product of two magnitudes--in themselves scalar quantities--yields a scalar product; this is exactly what we had set out to do. To summarize this:

> *The scalar product of two vectors is the product of the magnitude of one vector by the component of the magnitude of the second vector in the direction of the first.*

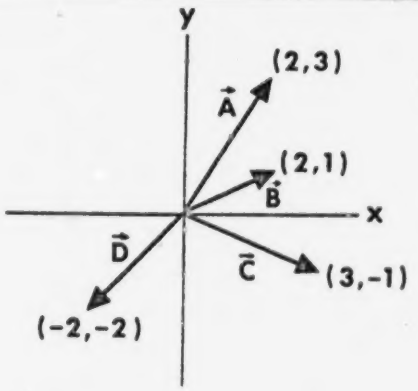
In working out the problems in the following section, you will find that you will be required to

(a) recognize a dot product as a scalar product;

(b) calculate the scalar product of two vectors placed at any angle to one another.

PROBLEMS

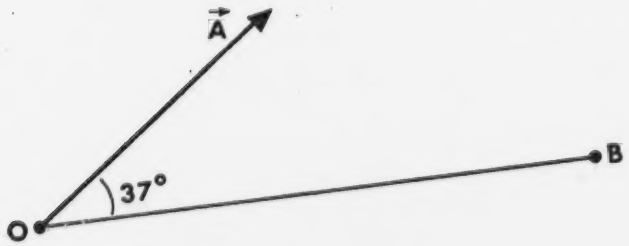
1.



Four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ are shown in the figure. The dot product $(\vec{A} + \vec{B})\cdot(\vec{C} - \vec{D})$ is equal to:

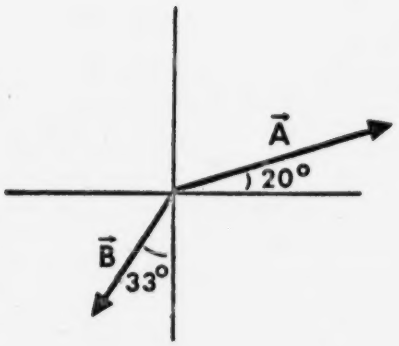
A. 16
B. 24
C. -8
D. -24

2.



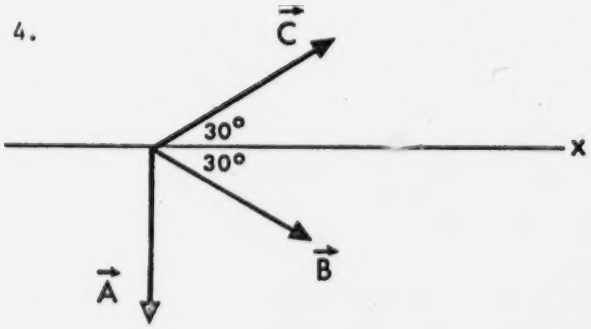
The vector $\vec{A}$ is 5 m long and makes an angle of 37° with a given line OB, as shown in the figure. Find the component of vector $\vec{A}$ in the direction of OB. (One significant digit in the answer is sufficient.)

3.



The two vectors $\vec{A}$ and $\vec{B}$ shown in the figure have magnitudes of 3 and 2, respectively. Their dot product is:

4.



Vector $\vec{A}$ of magnitude 4 is directed along the negative y-axis. The directions of vectors $\vec{B}$ and $\vec{C}$ are as shown, and their respective magnitudes are 4 and 5. The dot product $(\vec{A} + \vec{B})\cdot\vec{C}$ is equal to:

5. Four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ are in the xy-plane, i.e., each of the vectors has two components, one along the x-axis and the other along the y-axis. Find the dot product $(\vec{A} + \vec{B})\cdot(\vec{C} - \vec{D})$ in terms of the components of the four given vectors.

A. $(A_x + B_x)(C_x - D_x) + (A_y + B_y)(C_y - D_y)$
B. $(A_x - B_x)(C_x + D_x) + (A_y + B_y)(C_y + D_y)$
C. $(A_x + B_x)(C_x + D_x) + (A_y + B_y)(C_y + D_y)$
D. $(A_x - B_x)(C_x - D_x) + (A_y - B_y)(C_y - D_y)$

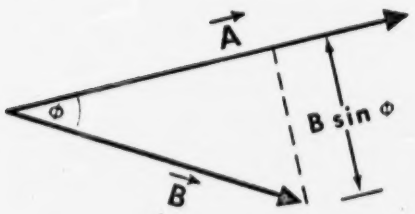
INFORMATION PANEL Cross (Vector) Product

OBJECTIVE

To apply the rules and conventions of vector multiplication in problems involving cross, or vector, products.

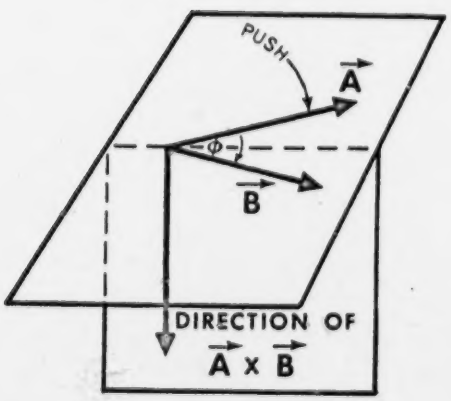
In keeping with our aim of defining mathematical operations so that they are most useful in physics, we will define the cross, or *vector*, product of two vectors $\vec{A}$ and $\vec{B}$ as

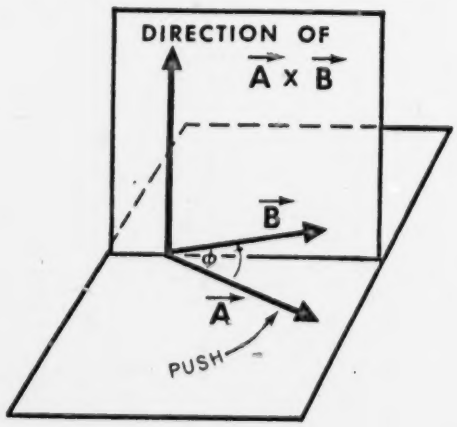
$$\vec{A}\times\vec{B} = AB\sin\phi$$



As illustrated in the diagram at the left, B sinϕ lies in the same plane formed by $\vec{A}$ and $\vec{B}$. By convention, however, the cross product $\vec{A}\times\vec{B}$ *is assigned a direction which is at right angles to this plane.* The *sense* of this new vector direction is also established by convention as follows:

First picture an axis passing perpendicularly through the AB plane at the point where the original vector arrows intersect. Next, curl the fingers of the right hand *around this axis in a direction such that they push vector* $\vec{A}$ into *vector* $\vec{B}$ *through the smaller angle between them. The outstretched thumb will now point in the direction assigned to the vector product* $\vec{A}\times\vec{B}$.





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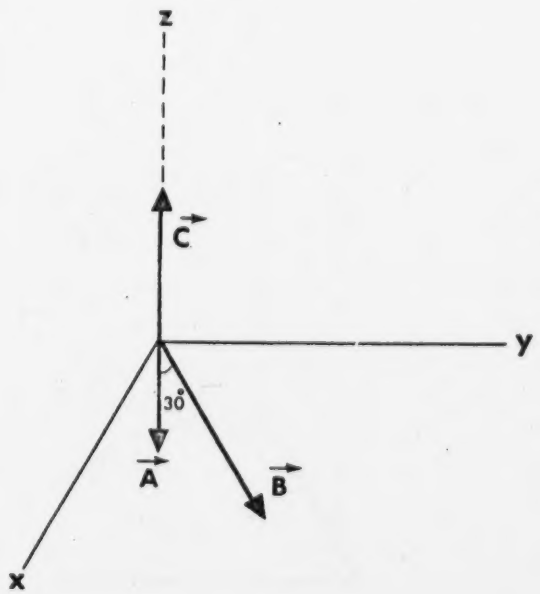
Thus, the resultant vector of a cross product has a direction different from either of the original vectors: it is always perpendicular to the plane in which the original vectors lie.

Students who encounter this direction convention for the first time are (rightfully) puzzled by its physical significance. It's easy enough, they say, to define an operation any way you like but in a definition like this, what does the resultant direction *mean*? What connection is there between the resultant vector of the cross product and the original vectors with regard to direction? Does this convention do any good in working out the relationships between things and events in the real environment?

The answers to these questions are forthcoming in your future study in physics. The fact that this definition has significance, that it does work in the environment, will become increasingly evident as you proceed with your studies.

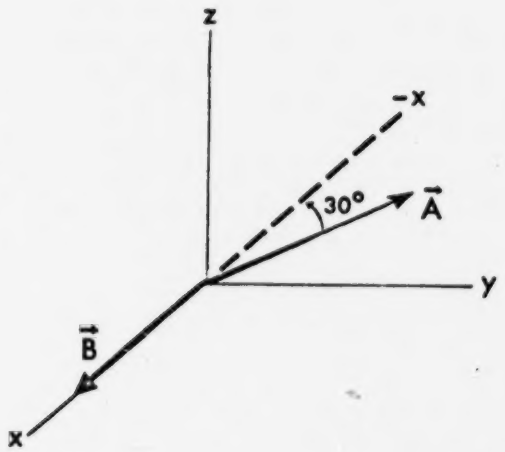
The problems incorporated in the following section deal with the mechanics of the mathematical process involved in obtaining cross products. These are introductory steps; the principles involved in them will be recalled to you when you reach subjects such as torque and angular momentum.

6.



Two vectors $\vec{A}$ and $\vec{B}$ are in the xy-plane. The magnitude of $\vec{A}$ is one unit and that of $\vec{B}$ is two units. $\vec{C}$ is another vector which has a magnitude of 2 units and lies along the positive z-axis. Find the product $(\vec{A} \times \vec{B}) \cdot \vec{C}$.

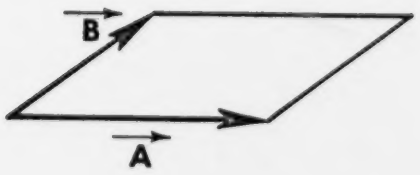
7.



Vector $\vec{A}$ lies on the xy-plane directed at an angle of 30° from the negative x-axis and has a magnitude 5. Vector $\vec{B}$ is along the x-axis and has a magnitude 4. The cross product (or vector product) $\vec{A}\times\vec{B}$ is a:

A. Scalar equal to 10
B. Vector of magnitude 10 in the positive z-direction
C. Vector of magnitude 10 in the negative z-direction.
D. Vector of magnitude 17.32 in the positive z-direction.

8.



Two vectors $\vec{A}$ and $\vec{B}$ define a parallelogram (see figure). The area of the parallelogram may be expressed as:

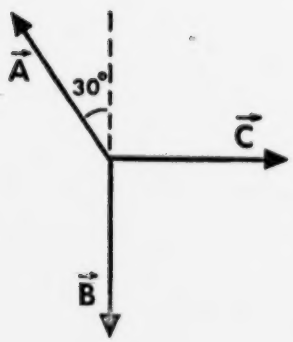
A. $|\vec{A}\times\vec{B}|$

B. $\vec{A}\cdot\vec{B}$

C. AB

D. $\left|\frac{\vec{A}+\vec{B}}{2}\right|$

9.



This figure shows three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, lying on a plane, each with a magnitude of 2 ft. The "triple" product $(\vec{A}\times\vec{B})\cdot\vec{C}$ is equal to:

A. a vector of magnitude 8 ft^3 and toward the left
B. zero
C. 8 ft^3
D. 4 ft^3

INFORMATION PANEL — Average Speed and Average Velocity

OBJECTIVE

To define and distinguish between the concepts of average speed and average velocity in problem-solving.

There is a simple little puzzle which sometimes trips up even knowledgeable people. It goes like this: A car makes a run from point A to point B, a distance of 240 miles, at a constant speed of 40 mi/hr. The driver then makes the return trip from B to A at a constant speed of 60 mi/hr. What was his average speed for the round trip?

Try this on your friends; you'll find most of them snap out the answer "50 mi/hr," without hesitating--but by now you know, of course, that this answer is incorrect. Since the trip from A to B requires more time, more "weight" should be given to 40 mi/hr than to 60 mi/hr in figuring the average. You *could* do it this way:

$$t_{AB} = \frac{\text{distance}}{\text{speed}} = \frac{240 \text{ mi}}{40 \text{ mi/hr}} = 6 \text{ hr}$$

$$t_{BA} = \frac{\text{distance}}{\text{speed}} = \frac{240 \text{ mi}}{60 \text{ mi/hr}} = 4 \text{ hr}$$

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The time "weighting" ratio is 6:4 or 3:2 as you can see, so the 40 mi/hr speed should get counted 3 times in the averaging process while the 60 mi/hr speed is counted twice. Thus,

$$\text{average speed} = \frac{40 + 40 + 40 + 60 + 60}{5} = 48 \text{ mi/hr}$$

However, it's much simpler to define average speed as *total distance* divided by *total time* so that,

$$\text{average speed} = \frac{(240 + 240) \text{ mi}}{(6 + 4) \text{ hr}} = \frac{480 \text{ mi}}{10 \text{ hr}} = 48 \text{ mi/hr}$$

We have been using the word "speed" rather than velocity advisedly; speed is a *scalar* quantity--actually the magnitude of velocity--while velocity is a *vector*. That is

$$\text{average speed} = \bar{v} = \frac{\text{distance (a scalar)}}{\text{time (a scalar)}} = \text{a scalar}$$

while

$$\text{average velocity} = \bar{\vec{v}} = \frac{\text{displacement (a vector)}}{\text{time (a scalar)}} = \text{a vector}$$

A bar above any symbol, e.g., v, will be used consistently to specify "average."

In solving the problems in this section, you will be asked to

(a) distinguish between average speed and average velocity;

(b) calculate average speed, given the distance and time;

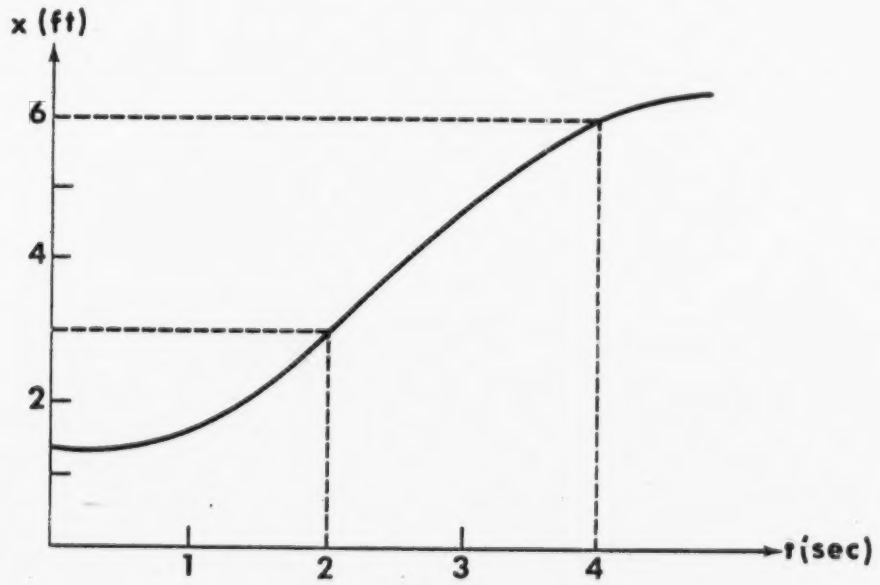
(c) calculate average speed from a distance time graph.

10. A student drives due east at 80 mi/hr for one hour, then drives at 60 mi/hr for another hour due north, and then returns to the starting point with a speed of 50 mi/hr. His average velocity ($\bar{\vec{v}}$) and average speed (v) over the entire journey are:

A. 0, 60 mi/hr
B. 0, 0
C. 30 mi/hr, 30 mi/hr
D. 60 mi/hr, 0

11. An automobile travels on a perfectly straight road and covers the distance between the 3rd and the 8th mileposts in 5 minutes. What is its average speed in meters/sec during this interval? (1 mi ≈ 1.61 km).

12. Find the average speed from the accompanying graph during the time interval from t = 2 to t = 4.



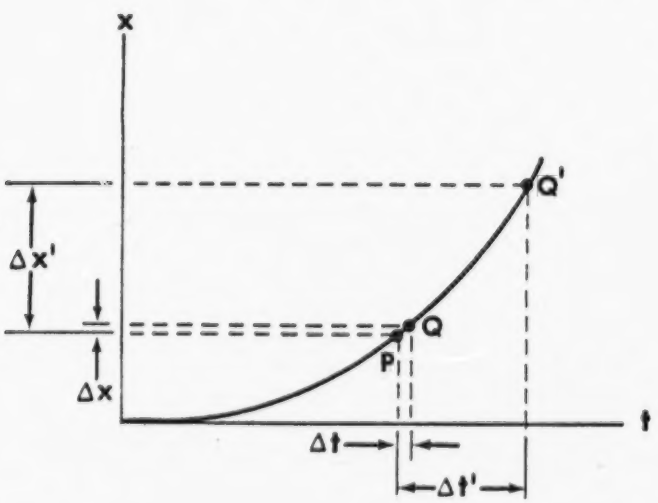
13. A car is travelling in a circular race track of radius 5.00 mi. It goes from a point A to a point B diametrically opposite to each other in 8.00 minutes. Find the magnitude of the average velocity ($\overline{\vec{v}}$) and average speed ($\overline{v}$) of the car for that run.

A. $\overline{\vec{v}}$ = zero; $\overline{v}$ = 118 mi/hr
B. $\overline{\vec{v}}$ = 75 mi/hr; $\overline{v}$ = 118 mi/hr
C. $\overline{\vec{v}}$ = 75 mi/hr; $\overline{v}$ = zero
D. $\overline{\vec{v}}$ = zero; $\overline{v}$ = zero

INFORMATION PANEL Instantaneous Velocity

OBJECTIVE

To define and apply the concept of instantaneous velocity to the solution of relevant problems.



The displacement-time graph of a moving particle appears at the left. As the particle moves from P to Q', its velocity increases as is evident from the fact that the slope of the curve increases. This means that the average velocity of the particle is different over various segments of its path. We can get a value for the average velocity from P to Q' using

$$\vec{v}_{PQ'} = \frac{\Delta\vec{x}'}{\Delta t'}$$

but we realize that this tells us very little about the actual velocity at specific points along the path *between* P and Q.

A valuable refinement of the concept of average velocity is obtained by imagining that Q' is brought very, very close to P, say to point Q. The displacement of the particle Δx for the tiny time interval Δt is also very small. The average velocity is

$$\vec{v}_{PQ} = \frac{\Delta\vec{x}}{\Delta t}$$

and for this short interval is more meaningful than the average velocity for the path P to Q' because Δt is too small to allow much of a change in velocity, that is, the slope of this short segment of the curve is nearly constant.

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Let us have Q approach P more and more closely. As this happens, Δt approaches zero as a limit and we indicate this as:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

Note that we have dropped the bar above the $\vec{v}$ because we are now defining $\vec{v}$ *at* point P, that is, as an *instantaneous velocity*. In the notation of the calculus, this becomes

$$\vec{v} = \frac{d\vec{x}}{dt}$$

Thus,

(a) *the instantaneous velocity $\vec{v}$ of a particle at any instant t is the first derivative of the coordinate position of the particle with respect to time, and*

(b) *the instantaneous velocity at any point on a displacement-time graph equals the slope of the tangent to the graph at that point.*

The problems in this set should help you reinforce your understanding of the concept of instantaneous velocity.

14. A particle moves in one dimension. Its position is described by the equation

$$x = \alpha(2t - t^2) + \beta t^3$$

where α and β are constants. Given that the particle's position changes from $x = 3$ m at $t = 1$ sec to $x = 16$ m at $t = 2$ sec, the magnitude of the particle's velocity in m/sec at $t = 3$ sec is:

15. A particle moves according to the equation $y = 16t^2$ (where y is given in feet). The velocity of the particle at t = 1 sec is given by:

A. 32 ft/sec
B. 16 ft/sec in the positive y-direction
C. 32 ft/sec in the positive y-direction
D. 16 ft/sec in the negative y-direction

16. The position of a particle, moving in a straight line is described by the equation

$$x = \alpha t + \beta t^2$$

where α and β are constants. Given that the particle's velocity changes from v = 14 m/sec at t = 2 sec to v = 26 m/sec at t = 4 sec. Find the magnitudes of α and β.

INFORMATION PANEL — Relative Velocity

OBJECTIVE

(a) To adopt a consistent system of symbols for describing relative motion;
(b) To develop the relationships needed to solve problems in relative velocity and to solve several such problems.

This INFORMATION PANEL aims to clarify the ideas about relative motion presented in your reading. As you should have gathered, we are concerned with finding the velocity of a particle relative to one coordinate system when we know (a) the velocity of this same particle in a different coordinate system and (b) the velocity of the second coordinate system relative to the first.

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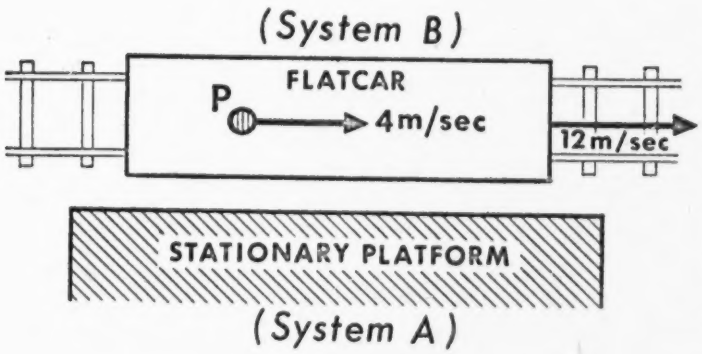
To simplify our approach, we shall restrict our discussion at this point to motion along a straight line, say the x-axis, retaining the vector directions (posicive to the right, negative to the left) but writing the relationships in scalar notation. In this way we can apply algebraic methods in addition and subtraction. Your reading shows rigorous proofs for assuming that vector extrapolation from the forthcoming treatment is quite valid.



In the diagram at the left we picture a flatcar (system B) moving to the right with a speed of 12 m/sec as measured by an observer on a stationary platform (system A). On the flatcar is a ball (the particle P) which is also moving to the right but with a constant speed of 4 m/sec *as measured by an observer standing on the moving flatcar.*

Now let's set up some symbols:

Let $+v_{BA}$ = speed of flatcar (B) relative to stationary observer (A) ----- 12 m/sec.

$+v_{PB}$ = speed of particle (P) relative to flatcar (B) ----- 4 m/sec.

v_{PA} = speed of particle (P) relative to platform (A) ----- direction as yet unknown but to be deduced from the sign of the final answer.

Imagine that you are the observer on the platform. How fast will *you* see the ball rolling? That is, what is the velocity of the ball relative to your point of view? Your intuitive answer will certainly be 16 m/sec to the right because you can see that the car's speed must add to the speed of the ball--12 m/sec + 4 m/sec = 16 m/sec.

Using the symbols above, this may be written as:

$$v_{PA} = v_{PB} + v_{BA}$$

or, the speed of P relative to A = the speed of P relative to B + the speed of B relative to A.

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Now note this: on the right side, the two inner subscripts are the same, both B's, and the two outer subscripts, P and A in that order are also the subscripts associated with the left side, in the same order. This pattern is always obtained when adding vectors in relative velocity situations if the sequence is correctly set up. It is often called the "domino rule."

DOMINO RULE

On the right side, the second subscript of the first term is the same as the first subscript of the second term; the two remaining subscripts are those of the term on the left side of the equation, in the same order.

Now consider this problem: car A going eastward at a speed of 50 mi/hr passes car B going westward on a parallel lane at a speed of 60 mi/hr. What is the speed of B as seen by the driver of A?

Let $+v_{AE}$ = speed eastward (+ direction) of A relative to the earth E.

$-v_{BE}$ = speed westward (- direction) of B relative to the earth E.

v_{BA} = speed of B as seen by (relative to) A, direction to be determined.

Symbolically,

$$v_{BA} = -v_{BE} + v_{AE}$$

To apply the domino rule, we must make the inner subscripts the same by reversing v_{AE} to become v_{EA}. The velocity of the earth relative to car A is merely the negative of the velocity of car A relative to the earth, hence since $v_{EA} = -v_{AE}$, we can rewrite the last expression as:

$$v_{BA} = -v_{BE} - v_{EA}$$

Before going ahead, check this for validity according to the domino rule; it is correct, as this check will show.

Substituting absolute values for the symbols, we have:

$$v_{BA} = - |60 \text{ mi/hr}| - |50 \text{ mi/hr}| = -110 \text{ mi/hr}$$

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So, the driver of car A sees car B go by in a negative (westward) direction at a speed of 110 mi/hr.

Checking a bit further: suppose you had wanted to know the speed of A as seen by B--and you can now see that the answer to this would be +110 mi/hr--you would set it up this way:

$$v_{AB} = v_{AE} - v_{BE}$$

in which you must reverse the subscripts in the second term on the right to get v_{EB}. Hence

$$v_{AB} = v_{AE} + v_{EB} \qquad \text{since } v_{EB} = -v_{BE}$$

so that v_{AB} = 50 mi/hr + 60 mi/hr = 110 mi/hr, the direction eastward as indicated by the positive answer.

The use of double subscripts for relative velocities works out nicely for motion in one dimension and should be consistently followed at least until you develop a "feel" for the concept. It is suggested that you apply the same methods in starting problems involving relative velocities in two dimensions, too, but that you prepare yourself to encounter symbolism that varies from problem to problem. The double subscript system, while invaluable in applying the domino rule, frequently is too cumbersome to carry through an entire problem as you will see.

In its most general form, applicable to conditions in two or more dimensions, the domino rule is valid for vector quantities and may be written:

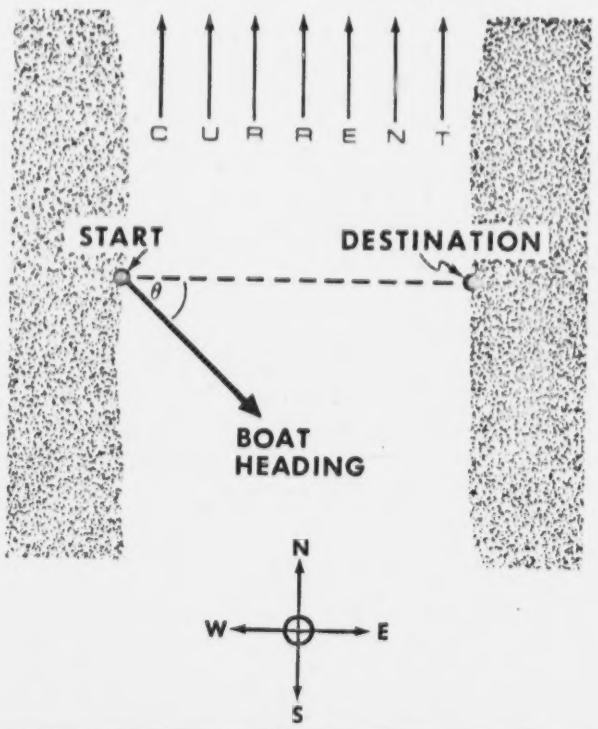
$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

The problems in this section require that you be able to

(a) apply the rules of relative velocity to the solution of typical problems;

(b) recognize that these rules apply to vector addition as well as algebraic addition.

17. A boat travels at the speed of 8 mi/hr relative to the water on a 1.5 mile wide river which flows due north at 10 mi/hr. A man starting from a point on the west bank wishes to reach the east bank at a point directly opposite to his point of start. Since the boat can not travel as fast as the stream, it is incapable of landing at the destination point; consequently the man must land downstream and run back. He runs at the rate of 6 mi/hr. Find the angle θ at which the man must head his boat to reach his destination in minimum time.



18. A river boat can travel in still water at the rate of 20 mi/hr. If the river current is 5 mi/hr, find the time for a round trip for the boat between two cities on the same bank 75 miles apart.

19. An aircraft carrier proceeds due north (N) at 30 knots and a truck aboard the carrier moves directly across it from port to starboard at 30 knots. The truck's velocity relative to the water is

A. $\sqrt{1800}$ sin45° knots; direction 45° S of E
B. $\sqrt{1800}$ sin45° knots; direction 45° N of E
C. $\sqrt{1800}$ knots; direction 45° S of E
D. $\sqrt{1800}$ knots; direction 45° N of E

20. Point B is 138 miles due east of point A. A plane must cover the distance from A to B in *one* hour (not counting take-off and landing times). If there is a steady 30-mile per hour wind blowing in a direction of 53° N of E, what is the velocity of the plane relative to the air?

A. 140 mi/hr; at 9.85° S of E
B. 122 mi/hr; at 11.3° S of E
C. 141 mi/hr; at 12.2° S of E
D. 140 mi/hr; at 11.3° S of E

21. A man can row a boat 4 mi/hr in still water. He wishes to cross a 2-mile wide river. If the speed of the river current is 3 mi/hr, find the shortest possible time required to cross the river.

[a] CORRECT ANSWER: Zero

The simplest way to solve this problem is to use the distributive law for rearranging the dot product; i.e.,

$$(\vec{A} + \vec{B})\cdot\vec{C} = \vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C} \qquad (1)$$

Using the fact that $\vec{A}\cdot\vec{C} = \vec{C}\cdot\vec{A} = AC\cos\theta$, where θ is the angle between $\vec{A}$ and $\vec{C}$, and similarly for $\vec{B}\cdot\vec{C}$, we obtain

$$(\vec{A} + \vec{B})\cdot\vec{C} = (4)(5)\cos 120° + (4)(5)\cos 60°$$

$$= 20(-0.5) + 20(0.5) = 0$$

In general, however, it will be simpler to work in terms of the vector components. Thus, for any two vectors $\vec{V}$ and $\vec{W}$,

$$\vec{V} + \vec{W} = (V_x\,\hat{\imath} + V_y\,\hat{\jmath}) + (W_x\,\hat{\imath} + W_y\,\hat{\jmath})$$

$$= (V_x + W_x)\,\hat{\imath} + (V_y + W_y)\,\hat{\jmath} \quad \text{a vector,} \qquad (2)$$

and

$$\vec{V}\cdot\vec{W} = (V_x\,\hat{\imath} + V_y\,\hat{\jmath})\,(W_x\,\hat{\imath} + W_y\,\hat{\jmath}) = V_xW_x + V_yW_y \quad \text{a scalar;} \qquad (3)$$

where the fact that $\hat{\imath}\cdot\hat{\imath} = 1 \times 1 \cos 0° = 1$, and
$\hat{\imath}\cdot\hat{\jmath} = 1 \times 1 \cos 90° = 0$ has been used.

For the problem at hand

$$(\vec{A} + \vec{B})\cdot\vec{C} = \vec{A}\cdot\vec{C} + \vec{B}\cdot\vec{C} = (A_xC_x + A_yC_y) + (B_xC_x + B_yC_y) \qquad (4)$$

The components for the vectors in this question are:

$A_x = 0$ $\qquad$ $A_y = -4$

$B_x = 4\cos 30° = 2\sqrt{3}$ $\qquad$ $B_y = -4\sin 30° = -2$

$C_x = 5\cos 30° = 5\sqrt{3}/2$ $\qquad$ $C_y = 5\sin 30° = 5/2$

Using these components, equation (4) becomes

$$(\vec{A} + \vec{B})\cdot\vec{C} = [0 + (-4)(5/2)] + [(2\sqrt{3})(5\sqrt{3}/2) + (-2)(5/2)]$$

$$= -10 + (15 - 5) = 0$$

The latter method was much more tedious in this particular problem. Yet in other cases it may be much simpler working with vector components.

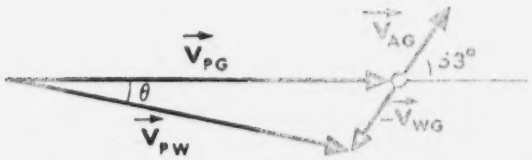
[a] CORRECT ANSWER: B

The problem supplies the velocity of wind relative to ground ($\vec{v}_{WG}$) directly. In addition, from the given positions of points A and B and the time requirements, we may compute the velocity of the plane relative to ground. Thus,

$$v_{PG} = \frac{138 \text{ mi}}{1 \text{ hr}} \hat{1} = 138 \text{ mi/hr east}$$

We must find $\vec{v}_{PW}$; so we write the relative velocity equations

$$\vec{v}_{PW} = \vec{v}_{PG} + \vec{v}_{GW} = \vec{v}_{PG} - \vec{v}_{WG} \qquad (1)$$



The x- and y-components of equation (1) respectively are:

$$v_{PW} \cos\theta = v_{PG} - v_{WG} \cos 53°$$
$$= v_{PG} - 0.6\, v_{WG} \qquad (2)$$

and

$$v_{PW} \sin\theta = 0 - v_{WG} \sin 53°$$
$$= -0.8\, v_{WG} \qquad (3)$$

Dividing equation (3) by (2) we get:

$$\tan\theta = \frac{0.8\, v_{WG}}{v_{PG} - 0.6\, v_{WG}} = -\frac{24 \text{ mi/hr}}{(138 - 18) \text{ mi/hr}} = -\frac{24}{120} = -0.2$$

and $\theta = -11.3°$ (4)

The negative angle indicates that $\vec{v}_{PW}$ is directed below the positive x-axis; i.e., 11.3° S of E.

Now $\sin(-11.3°) = -0.196$ so from (3) we get

$$v_{PW} = \frac{-24 \text{ mi/hr}}{-0.196} = 122 \text{ mi/hr}$$

[a] CORRECT ANSWER: 50 m/sec

The expressions for the position and velocity of the particle are:

$$x = \alpha(2t - t^2) + \beta t^3 \qquad (1)$$

$$v \equiv \frac{dx}{dt} = \alpha(2 - 2t) + 3\beta t^2 \qquad (2)$$

Substituting the given data in equation (2) we find:

at $t = 1$, $\qquad x(1) = 3 = \alpha(2 - 1) + \beta \qquad (3)$

at $t = 2$, $\qquad x(2) = 16 = \alpha(4 - 4) + 8\beta \qquad (4)$

Solving equations (3) and (4) simultaneously, we get

$$\alpha = 1 \qquad \text{and} \qquad \beta = 2$$

Substituting these values of α and β in equation (2) we find, for $t = 3$

$$v(3) = (1)(2 - 6) + (3)(2)(9) = 50 \text{ m/sec}$$

TRUE OR FALSE? Equation (2) in the solution above is the first derivative of velocity with respect to time.

[b] CORRECT ANSWER: C

The small angle between the two vectors is $\theta = 90° + 60° = 150°$. The cross product of two vectors is a vector of magnitude

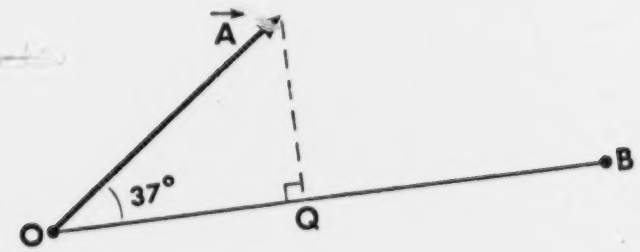
$$C \equiv |\vec{A}\times\vec{B}| = AB \sin\theta$$

In this case,

$$C = (5)(4) \sin 150° = 20(1/2) = 10$$

The right-hand rule is used to determine the direction of $\vec{C}$. We curl our right-hand fingers from the direction of $\vec{A}$ to that of $\vec{B}$ through the smaller angle between them. The vector $\vec{C}$ points towards the direction indicated by the extended thumb, which is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. For the present problem, $\vec{C}$ is directed towards the negative z-axis.

[a] CORRECT ANSWER: 4 meters



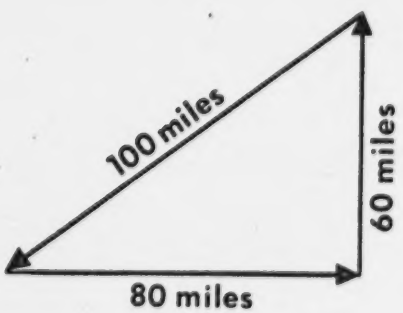
The component of vector $\vec{A}$ in the direction OB is obtained by dropping a perpendicular line from the tip of vector $\vec{A}$ to line OB as shown in the figure. The required component, OQ, is given by

$$OQ = A_B = A \cos 37°$$

$$= 5 \text{ m} \times 0.8 = 4 \text{ m}$$

[b] CORRECT ANSWER: A

You may recall that the average velocity is defined as the displacement during a time interval divided by that time interval. In this case, since the car returns to its point of origin, the total displacement is zero, and consequently the average velocity is zero.



On the other hand, the average speed is simply the total distance covered during a time interval divided by that time interval. Here the final leg of the trip would take him two hours because the distance in the last leg of the trip is 100 miles. Hence, the average speed for the entire trip is

$$\bar{v} = \frac{80 + 60 + (50 \times 2)}{1 + 1 + 2} = 60 \text{ mi/hr}$$

TRUE OR FALSE? The average speed of a body is the magnitude of its average velocity.

[a] CORRECT ANSWER: 8 hr

Let v_{BW} = speed of boat with respect to water

v_{WE} = velocity of current with respect to Earth, downstream

v_{BE} = velocity of boat with respect to Earth, downstream

and d = distance between the two cities

Inerefore,

$$\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$$

Let us assume that the boat first makes the trip downstream and returns upstream.

The time for the downstream leg of the trip is

$$t_d = \frac{d}{v_{BE}}$$

For the return leg, the velocity of the boat with respect to Earth, upstream, is taken to be $\vec{v}_{BE}'$, where

$$\vec{v}_{BE}' = \vec{v}_{BW} + (-\vec{v}_{WE})$$

The corresponding time for the return trip is

$$t_u = \frac{d}{v_{BE}'}$$

The total time for the round trip is

$$t = t_d + t_u = \frac{d}{v_{BE}} + \frac{d}{v_{BE}'}$$

or

$$t = \frac{d}{v_{BW} + v_{WE}} + \frac{d}{v_{BW} - v_{WE}}$$

$$= \frac{75 \text{ mi}}{20 \text{ mi/hr} + 5 \text{ mi/hr}} + \frac{75 \text{ mi}}{20 \text{ mi/hr} - 5 \text{ mi/hr}}$$

$$= 8 \text{ hr}$$

[a] CORRECT ANSWER: A

Let

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$$

$$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j}$$

$$\vec{C} = C_x\,\hat{i} + C_y\,\hat{j}$$

and

$$\vec{D} = D_x\,\hat{i} + D_y\,\hat{j}$$

where (A_x, B_x, C_x, D_x) and (A_y, B_y, C_y, D_y) are the projections of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ on the x- and y-axis, respectively. $\hat{i}$ and $\hat{j}$ are two mutually perpendicular vectors of unit magnitude, with directions along the x- and y-axis, respectively.

Since the sum, and also the difference of two vectors are vectors, we may say

$$\vec{A} + \vec{B} = \vec{R} \qquad (1)$$

and

$$\vec{C} - \vec{D} = \vec{Q} \qquad (2)$$

Therefore,

$$\vec{A} + \vec{B} = \hat{i}(A_x + B_x) + \hat{j}(A_y + B_y) = \hat{i}\,R_x + \hat{j}\,R_y$$

and

$$\vec{C} - \vec{D} = \hat{i}(C_x - D_x) + \hat{j}(C_y - D_y) = \hat{i}\,Q_x + \hat{j}\,Q_y$$

By equating the components from both sides of equations (3) and (4), we get

$$\begin{aligned} A_x + B_x &= R_x \\ A_y + B_y &= R_y \end{aligned} \qquad (5)$$

and

$$\begin{aligned} C_x - D_x &= Q_x \\ C_y - D_y &= Q_y \end{aligned} \qquad (6)$$

Now the dot product of $\vec{R}$ and $\vec{Q}$ is a scalar by definition and is given by

$$\vec{R}\cdot\vec{Q} = (\hat{i}\,R_x + \hat{j}\,R_y)\cdot(\hat{i}\,Q_x + \hat{j}\,Q_y)$$

$$= R_xQ_x + R_yQ_y \qquad (7)$$

next page

continued

by the fact that

$$\mathbf{i}\cdot\mathbf{j} = (1)(1)\cos 90° = 0$$

and

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = (1)(1)\cos 0° = 1$$

Hence,

$$(\vec{A} + \vec{B})\cdot(\vec{C} - \vec{D}) = \vec{R}\cdot\vec{Q} = R_xQ_x + R_yQ_y$$

Substitution for R_x, R_y, Q_x and Q_y from equation (5) and (6) into (7) leads to

$$(\vec{A} + \vec{B})\cdot(\vec{C} - \vec{D}) = (A_x + B_x)(C_x - D_x) + (A_y + B_y)(C_y - D_y)$$

which is obviously a scalar.

TRUE OR FALSE? The final product in the problem above is a vector quantity.

[a] CORRECT ANSWER: B

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time for the run}}$$

$$\vec{\bar{v}} = \frac{\Delta\vec{s}}{\Delta t}$$

In this problem,

(i) $\Delta s = 2R = 10$ mi where R is the radius of the track

and $\Delta t = 8$ min

Thus the magnitude of average velocity is

$$\bar{v} = \frac{10 \text{ mi}}{8 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 75 \text{ mi/hr}$$

(ii) $$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time for the run}}$$

$$= \frac{\pi R}{\Delta t}$$

$$= \frac{3.14 \times 5 \text{ mi}}{8 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 118 \text{ mi/hr}$$

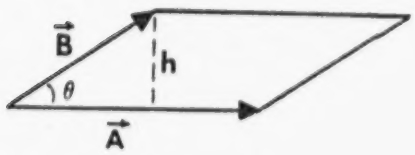
TRUE OR FALSE? We are consistently using $\bar{v}$ to represent average speed.

[a] CORRECT ANSWER: A

The area of the given parallelogram is S = Ah. However,

$$h = B \sin\theta; \quad \text{so,}$$

$$S = AB \sin\theta = |\vec{A}\times\vec{B}|$$



Later on in this course you will come across certain situations in which an area is defined not only by its magnitude but also by a certain direction. Thus, in physics, area is considered a vector.

The direction of the area of a curved surface is, by convention, taken to be the direction of the outward normal to the surface at the point in question.

For plane surfaces, the direction of the area is constant. Again by convention, the positive direction of a plane surface is taken to be the direction of the extended right-hand thumb, with the other fingers curled around the area in a counterclockwise sense. Thus, the area for the given parallelogram could be written as

$$\vec{S} = \vec{A}\times\vec{B}$$

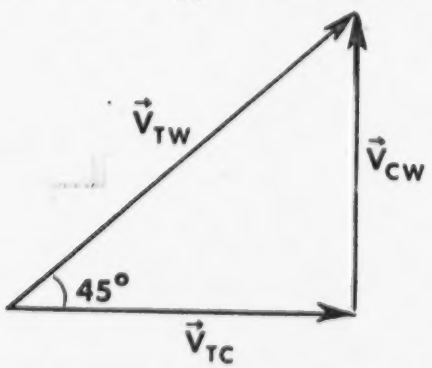
[b] CORRECT ANSWER: D

We will use the following subscripts: W for water, C for carrier, and T for truck. The given quantities are

$$\vec{v}_{CW} = 30 \text{ knots N} \quad \text{and} \quad \vec{v}_{TC} = 30 \text{ knots E}$$

We must compute $\vec{v}_{TW}$. These three velocities are related by the equation

$$\vec{v}_{TW} = \vec{v}_{TC} + \vec{v}_{CW}$$



The addition of the two velocities on the right-hand-side is best done geometrically, as shown in the figure. From the isosceles right triangle formed, $\vec{v}_{TW}$ is easily computed. Thus,

$$\vec{v}_{TW} = \sqrt{(30)^2 + (30)^2}$$

$$= \sqrt{1800} \text{ at } 45° \text{ N of E}$$

[a] CORRECT ANSWER: $\alpha = 2$, $\beta = 3$

The expression for the velocity is

$$v = \frac{dx}{dt} = \frac{d}{dt}(\alpha t + \beta t^2)$$

$$= \alpha + 2\beta t \qquad (1)$$

Substituting the given data in equation (1) we find

at $t = 2$ sec

$$v(2) = 14 \text{ m/sec} = (\alpha + 4\beta) \qquad (2)$$

and at $t = 4$ sec

$$v(4) = 26 \text{ m/sec} = (\alpha + 8\beta) \qquad (3)$$

Solving for α and β from equations (2) and (3) we obtain

$$\alpha = 2 \quad \text{and} \quad \beta = 3$$

The units of α are m/sec and β are m/sec^2 arrived at by inspection of any one of the equations.

TRUE OR FALSE? The constant β has the dimension of acceleration.

[b] CORRECT ANSWER: 26.8 m/sec

From the computed displacement (8 - 3 = 5 mi) and the elapsed time (5 min), we find the average speed

$$\bar{v} = (5 \text{ mi})/(5 \text{ min}) = 1 \text{ mi/min} = \frac{1610}{60}\frac{\text{m}}{\text{sec}} = 26.8 \text{ m/sec}$$

[c] CORRECT ANSWER: -4.8

The dot product is given by $AB\cos\theta$, where $A = 3$, $B = 2$, and $\theta = 143°$. Since $\cos 143° = \cos(180° - 37°) = -\cos 37° = -0.8$, the dot product becomes:

$$3 \times 2 \times (-0.8) = -4.8$$

[a] CORRECT ANSWER: 30°

Let us first solve the problem symbolically. The symbols used are:

$v_{RE} = v_o$ = speed of the river

$v_{BW} = v_b$ = speed of the boat

$v_{ME} = u$ = the speed at which the man runs

d = width of the river

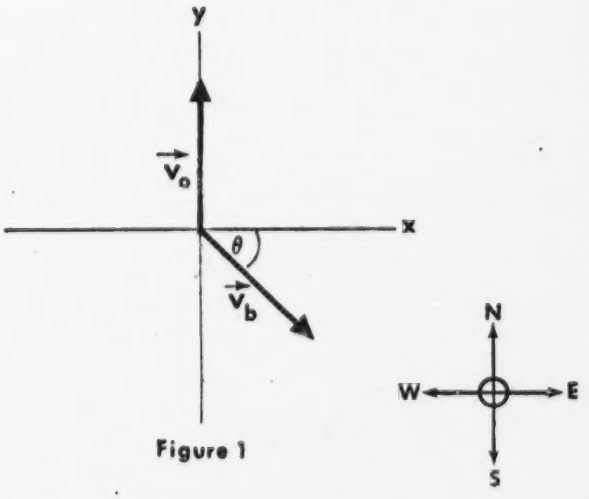


Figure 1

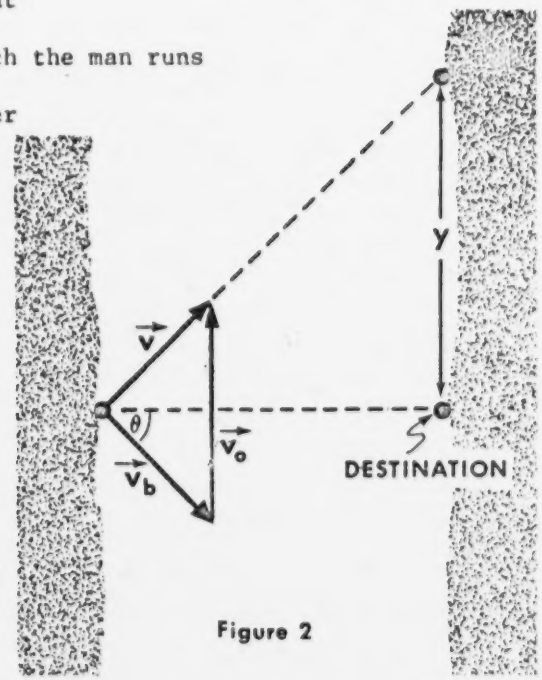


Figure 2

Let θ be the angle at which the boat heads as shown in Figure 1. The components of the resultant velocity v_x and v_y are given by

$$v_x = v_b \cos\theta \tag{1}$$

and

$$v_y = v_o - v_b \sin\theta \tag{2}$$

Therefore, the time t_1 the boat takes to cross the river is

$$t_1 = \frac{d}{v_x} = \frac{d}{v_b \cos\theta} \tag{3}$$

The distance, y, the man lands downstream from his destination is given by (see Figure 2)

$$y = v_y t_1$$

$$= (v_o - v_b \sin\theta)\, t_1 \tag{4}$$

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continued

The time, t_2, required to run back to his destination is

$$t_2 = \frac{y}{u}$$

Substitution of values for y and t_1 from equations (4) and (3) yields

$$t_2 = \frac{(v_o - v_b \sin\theta)\, d}{u v_b \cos\theta} \qquad (5)$$

To obtain the total time T, we add t_1 and t_2. Therefore,

$$T = t_1 + t_2 = \frac{d}{v_b \cos\theta} + \frac{(v_o + v_b \sin\theta)\, d}{u v_b \cos\theta}$$

$$= \frac{d}{v_b}\left[1 + \frac{v_o}{u}\right]\frac{1}{\cos\theta} - \frac{d}{u}\tan\theta \qquad (6)$$

In expression (6), all the quantities on the right-hand-side are constants, except θ. That is to say, T is a function of θ only. To minimize the function, we differentiate T with respect to θ and equate to zero.

$$\frac{dT}{d\theta} = \frac{d}{v_b}\left[1 + \frac{v_o}{u}\right] \sec\theta \tan\theta - \frac{d}{u}\sec^2\theta = 0$$

or

$$= \frac{d}{v_b}\left[1 + \frac{v_o}{u}\right] \sin\theta \sec^2\theta - \frac{d}{u}\sec^2\theta = 0$$

Solving for $\sin\theta$, we get

$$\sin\theta = \frac{v_b}{u + v_o} \qquad (7)$$

Substitution of numerical values in equation (7) leads to

$$\sin\theta = \frac{1}{2}$$

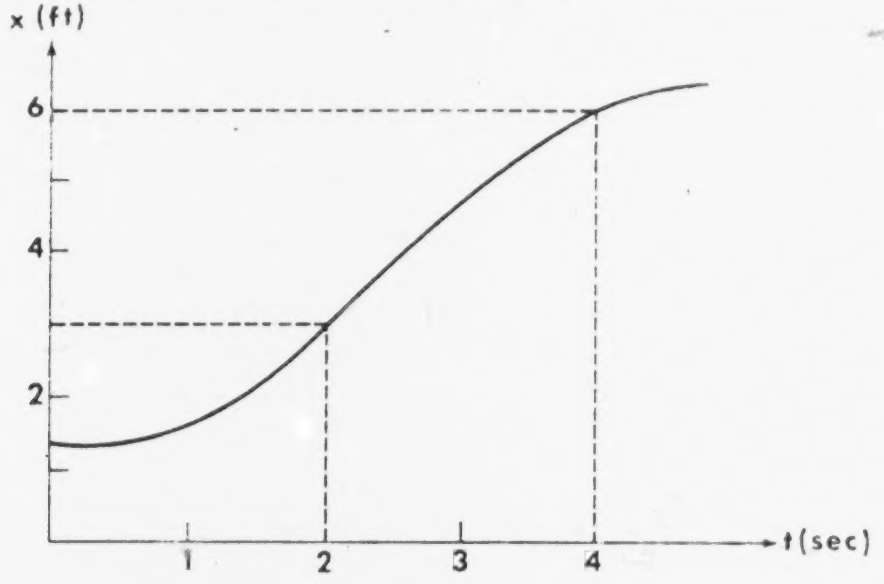
so

$$\theta = 30°$$

TRUE OR FALSE? In this problem, it is impossible for the man to make the entire trip to his destination by water.

[a] CORRECT ANSWER: 1.5 ft/sec

Since we are interested in the average *speed*, all we have to know is the magnitude of the displacement during the specified time interval.



Looking at the x versus t graph, we see that during the two seconds from $t = 2$ to $t = 4$ the particle has moved from $x_i = 3$ ft to $x_f = 6$ ft. Hence

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{6 - 3 \text{ ft}}{4 - 2 \text{ sec}} = \frac{3}{2} \frac{\text{ft}}{\text{sec}}$$

Note that the path the particle followed in moving from $x_i = 3$ to $x_f = 6$ is completely irrelevant to this question. In other words, the average velocity is the *constant* slope of the straight line joining the two end positions.

[b] CORRECT ANSWER: C

The velocity is the time derivative of the displacement. From $y = 16t^2$ ft we get

$$v = \frac{dy}{dt} = 32t \frac{\text{ft}}{\text{sec}}$$

At $t = 1$ sec, $v = 32$ ft/sec. Now, $y \geq 0$ for any t and, furthermore, y increases with increasing t. Therefore, the velocity is directed in the positive y-direction.

[a] CORRECT ANSWER: 2 units

The magnitude of the vector product $\vec{A}\times\vec{B}$ is

$$|\vec{A}\times\vec{B}| = AB\ \sin 30^\circ = (2)(1)\ \sin 30^\circ = 1$$

and it is in the direction of the positive z-axis. Now the dot product between $(\vec{A}\times\vec{B})$ and $(\vec{C})$ is given by

$$(\vec{A}\times\vec{B})\cdot\vec{C} = |\vec{A}\times\vec{B}|\,|\vec{C}|\ \cos\theta$$

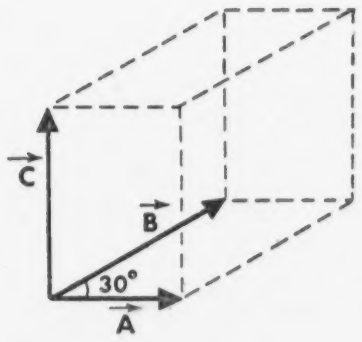
where θ is the angle between two vectors $\vec{A}\times\vec{B}$ and $\vec{C}$. In this case the vectors $\vec{A}\times\vec{B}$ and $\vec{C}$ are parallel; then $\theta = 0^\circ$.

Thus,

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (1)(2)\ \cos 0^\circ$$

$$= 2 \text{ units}$$

Note that geometrically this product gives the volume of the parallelepiped formed by these vectors as shown below.



TRUE OR FALSE? Two vectors of different finite length may have a cross product of zero length.

[a] CORRECT ANSWER: 30 min

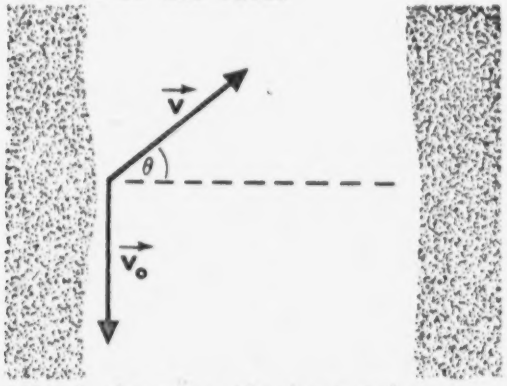
Let

$v_{WE} = v_o$ = speed of river current

$v_{BW} = v$ = speed of boat in still water

θ = the angle the boat makes with a line perpendicular to the direction of river current

d = width of the river



For an arbitrary angle θ the time the boat takes to cross the river is

$$t = \frac{d}{v \cos\theta} \qquad (1)$$

Equation (1) shows that t is a function of θ. In order to find the minimum value of t we differentiate t with respect to θ and equate to zero.

$$\frac{dt}{d\theta} = \frac{d}{v} \sec\theta \tan\theta = 0$$

or

$$\frac{\sin\theta}{\cos^2\theta} = 0$$

and this yields

$$\theta = 0°$$

Substitution of numerical values in equation (1) yields

$$t = \frac{2 \text{ mi}}{4 \text{ mi/hr}} = 0.5 \text{ hr} = 30 \text{ min}$$

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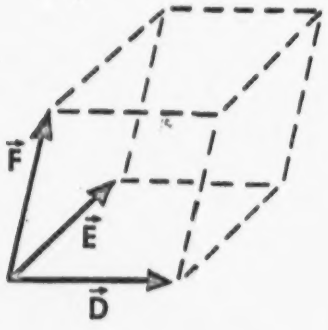
It should be noted that we could have arrived at the same answer by inspection of equation (1). In order that t be a minimum, $\cos\theta$ must be a maximum, i.e.,

$$\cos\theta = 1 \quad \text{or} \quad \theta = 0$$

TRUE OR FALSE? As indicated above, $\sin\theta/\cos^2\theta = 0$ is a standard trigonometric identity.

[a] CORRECT ANSWER: B

The triple product of any three vectors in space gives the volume of the parallelepiped formed by them (see figure).



Let us see why this is true. In the figure, if $\vec{D}$ and $\vec{E}$ are taken to lie on the plane of the page, then the cross product $\vec{G} \equiv \vec{D}\times\vec{E}$ is the "area" of the base of the parallelepiped, directed out of the page. The dot product $\vec{G}\cdot\vec{F}$ is equal to the magnitude of $\vec{G}$ (area of the base) times the projection of $\vec{F}$ on $\vec{G}$ (height of the parallelepiped). But this product is indeed equal to the volume of the parallelepiped.

TRUE OR FALSE? The volume turns out to be zero because the three original vectors are in the same plane.

[b] CORRECT ANSWER: B

There should be no doubt in your mind that the component method is the most appropriate one for solving this problem. Using the given components, we find

$$(\vec{A} + \vec{B})\cdot(\vec{C} - \vec{D}) = [(A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}]\cdot[(C_x - D_x)\hat{\imath} + (C_y - D_y)\hat{\jmath}]$$

$$= (A_x + B_x)(C_x - D_x) + (A_y + B_y)(C_y - D_y)$$

$$= (2 + 2)[3 - (-2)] + (3 + 1)[-1 - (-2)] = 24$$

TRUE OR FALSE? In this solution, the individual x- and y-components of each of the vectors were added before finding the dot product.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Problem Solving Procedures

OBJECTIVE

To solve a variety of problems dealing with motion in one dimension; to recognize that certain types of problems which appear two-dimensional may be reduced to a one-dimensional approach.

It would be of great advantage to you at this point to read through the following material carefully in preparation for the problem solving session you are about to begin. Although we are setting the procedures forth in the form of simple rules, you should be aware that you may vary them to suit yourself as long as they do not lead you into blind alleys.

(a) Some problems may have the desired quantity buried in a mass of verbiage. It's a good idea to list the given data and then establish for yourself just what the problem is seeking.

(b) Decide which equation or equations are needed to help you solve the particular problem. Write them down.

(c) In most problems, vector equations are called for since direction is important. Even the one-dimensional equations of motion, although written in scalar form, are actually vector equations. *The direction of each quantity must be established before attempting to use these equations.* The choice of a preferred positive direction is completely arbitrary, but in the beginning you are urged to follow the rules outlined below, at least until you develop the confidence you need to vary them.

(1) On the x-axis, the direction to the *right* is to be taken as *positive* even though it may appear inconvenient at times.

(2) On the y-axis, the *upward* direction on the page is to be taken as *positive*. Specifically, in problems dealing with objects moving under the influence of gravity, the direction away from the ground is to be taken as positive.

(3) In an equation written in scalar form, a vector quantity directed along the chosen positive direction should be taken as positive. An oppositely-directed vector quantity should be entered in the equation with a negative sign.

next page

continued

(4) Problems should be solved in symbolic or literal form with substitution of numbers being delayed until the very last step. This procedure permits substantial simplifications before you have to deal with numbers. In addition, a quick dimensional check is much more easily performed with the equation in symbolic form enabling you to detect unsuspected algebraic errors.

PROBLEMS

1. A rocket ascends with an effective, resultant constant acceleration of 64 ft/sec^2. Five seconds after lift-off, however, its engine shuts off. What is the highest altitude it reaches?

2. A particle is dropped from rest at a certain height. Some time later is strikes the ground with a speed of 16 ft/sec. How long was its time of fall?

3. A stone is thrown vertically upwards with an initial speed v_o. After 3.00 seconds its speed has decreased to 40.0 m/sec but it is still ascending. What was its initial speed v_o? (Neglect air resistance.)

4. A stone is thrown vertically downwards from the top of a cliff 100 ft high. Two seconds later the stone hits the ground at the bottom of the cliff. What was the initial speed of the stone?

5. A car starting from rest accelerates at 15 ft/sec^2. Ten seconds after starting, however, the driver applies the brakes. The braking deceleration of the car is 25 ft/sec^2. Find the total distance travelled by the car.

6. A batted baseball leaves the bat in a vertical upward direction. At the time of contact the bat was 5.0 ft above the ground. Four (4.0) seconds later the ball lands on the plate. What is the highest point above the ground that the ball reached?

7.

Distance of Fall (m)	Time (sec)
0.000	0.00
0.708	0.50
1.835	1.0
3.377	1.5
5.340	2.0

The table gives possible data from an experiment performed on the surface of the moon to measure the acceleration of gravity g_m on the moon. From the analysis of these data, the value of g_m is:

8. A student sees a ball fly up and then back down past a window 26.00 ft high. If the total time the ball is in sight is 2.00 sec, find the height above the window to which the ball rises.

9. The distance between point A and wall B is 3000 ft. A car can develop a maximum acceleration of 15 ft/sec^2. The maximum deceleration that the brakes can provide is 30 ft/sec^2. The driver of the car wants to reach the wall B in the shortest possible time, starting from rest at point A. He uses the full accelerating capacity of the car. What is the shortest distance from B at which he must apply the brakes if he is to avoid crashing into the wall?

10. With what initial speed must a ball be thrown vertically upward in order to rise to a maximum height of 49 ft?

11. The distance between point A and point B is 4000 ft. A car can develop a maximum acceleration of 10.0 ft/sec^2. The maximum deceleration that the brakes can provide is 30.0 ft/sec^2. The driver of the car, starting from rest at point A wishes to come to stop at point B. Find the shortest possible time for the trip.

INFORMATION PANEL — Equation Summary for Projectile Motion

OBJECTIVE

(a) To analyze projectile trajectories into vertical and horizontal velocities and displacements;

(b) To solve problems dealing with projectile altitude, time of flight, and instantaneous displacement.

The path or *trajectory* of a projectile is the resultant of a combination of horizontal motion with constant velocity and vertical motion with constant acceleration, the combining process taking place instant by instant as the projectile moves.

A run down of the standard symbolism we shall use is probably the most useful information we can provide in this Panel. We have set it up in chart form for ready reference throughout the segment.

v_o --- actual initial velocity of the projectile at some angle to a reference plane, usually the ground (horizontal)

θ_o --- angle made by vector v_o with horizontal plane, called *angle of departure*

v_{ox} --- horizontal component of v_o

v_{oy} --- vertical component of v_o

next page

continued

$$v_{ox} = v_o \cos\theta_o \text{ ----- (1)}$$

$$v_{oy} = v_o \sin\theta_o \text{ ----- (2)}$$

v_y --- instantaneous vertical velocity at any time t

v_x --- instantaneous horizontal velocity at any time t

$$v_x = v_o \cos\theta_o \text{ ----- (3)}$$

$$v_y = v_o \sin\theta_o - gt \quad (4)$$

v --- resultant or actual velocity at any time t

θ --- angle between resultant instantaneous velocity and horizontal plane

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1} \frac{v_y}{v_x}$$

x --- position coordinate referred to horizontal reference axis

y --- position coordinate referred to vertical reference axis

$$x = (v_o \cos\theta_o)\, t$$

$$y = (v_o \sin\theta_o)\, t - \frac{1}{2} gt^2$$

Here are a few other useful facts to bear in mind when working on projectile problems.

*Although the vertical and horizontal components of the motion are independent of one another, they do have one thing in common: *the time of flight is the same for both.*

*When two (or more) projectiles moving in the same plane collide in midair, they have the same x- and y-coordinates at the instant of collision but they *do not necessarily have the same flight times.*

The following problems require that you be sufficiently conversant with the equations and concepts of one-dimensional motion to be able to

(a) calculate the return (impact) velocity of a projectile in a parabolic trajectory;

next page

continued

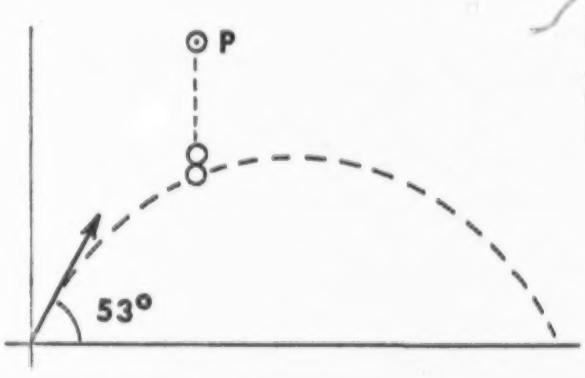
(b) determine maximum height of ascent and/or ascent time;

(c) find the various parameters which obtain when two projectiles collide in midair;

(d) determine flight time and range for any angle of departure;

(e) determine angle of departure for maximum range.

12. A ball is shot from the origin with an initial velocity of 17.5 m/sec at 53° above the horizontal. At the same instant, a second ball is released from a point P shown in the figure. In one second (1 sec), the two balls collide in mid-air. What is the altitude of the point P? (HINT: Consider the two balls as point particles.)



A. 5.6 m
B. 20 m
C. 14 m
D. 10 m

13. A ball is projected with a velocity $\vec{v}$ = 30 ft/sec, at 30° above the ground level of a horizontal field. What is the ball's *vertical* speed when it hits the ground?

A. 15 ft/sec
B. 30 ft/sec
C. 26 ft/sec
D. 32 ft/sec

14. A cannon shell is shot from ground level at an angle of 30° above the horizontal with a speed of 640 ft/sec. Neglecting air resistance, what is the highest altitude it will reach?

A. 1600 ft
B. 3200 ft
C. 800 ft
D. 2770 ft

15. Referring to the situation in the preceding question, how long does it take the shell to reach its maximum height?

A. $\sqrt{20}$ sec
B. 20 sec
C. $\sqrt{10}$ sec
D. 10 sec

16. In Problem 14, when the shell is at its maximum height, its horizontal speed and horizontal position, respectively, are:

A. 320 ft/sec; 1600 ft
B. 554 ft/sec; 1600 ft
C. Zero; 5540 ft
D. 554 ft/sec; 5540 ft

17. A projectile with an initial velocity of 40 m/sec is fired at an angle of 30° with the horizontal. Simultaneously another projectile is fired vertically upward from a point at a distance 60 m from the launching site of the first projectile. Assuming that both projectiles move in the same plane, find the initial velocity of the second projectile that will insure collision.

18. A cannon can project a shell with initial speed of 500 ft/sec. Assume that the shell leaves the cannon at ground level, and that air resistance may be neglected. Find the maximum range of the cannon in miles.

19. A ball is projected from ground level with an initial velocity of 14 m/sec at 45° above the horizontal. What is its range?

20. True or false? The relation between the maximum height y_{max}, and the range R, of a projectile having initial speed v_o and angle of projection θ_o is

$$\frac{y_{max}}{R} = \frac{1}{4} \tan \theta_o$$

[a] CORRECT ANSWER: 67 ft

Let t_1 be the time of ascent. Then,

$$h = h_o + v_o t_1 - (1/2)gt_1^2 \qquad (1)$$

In (1), h is the maximum height attained, which we wish to calculate, $h_o = 5$ ft, and v_o is the unknown initial upward speed.

In its descent, the ball starts with zero speed at an initial height h, its final height after time t_2 being zero. Thus,

$$0 = h - (1/2)gt_2^2 \qquad (2)$$

Since we have four unknowns (t_1, t_2, v_o and h), we must use two more equations. One of them can be the velocity equation for the ascent, with the final velocity equal to zero, i.e.

$$0 = v_o - gt_1 \qquad (3)$$

Finally, the sum of the two times gives the total time of flight:

$$t_1 + t_2 = t \qquad (4)$$

Equation (3) gives $v_o = gt_1$, which when substituted into (1) yields

$$h = h_o + gt_1^2 - (1/2)gt_1^2 = h_o + (1/2)gt_1^2 \qquad (5)$$

We solve (4) for $t_1 = t - t_2$ and substitute this into (5) to obtain

$$h = h_o + (1/2)g\,(t^2 + t_2^2 - 2tt_2) \qquad (6)$$

From equations (2) and (6) we get

$$(1/2)gt_2^2 = h_o + (1/2)gt^2 + (1/2)gt_2^2 - gtt_2$$

or

$$t_2 = \frac{2h_o + gt^2}{2gt} \qquad (7)$$

Substituting the values $h_o = 5$ ft, $g = 32$ ft/sec^2 and $t = 4$ sec yields

$$t^2 = \frac{2(5) + 32(4^2)}{2(32)(4)} = 2.04 \text{ sec}$$

Finally, substitution of t_2 in (2) gives

$$h = (1/2)(32)(2.04)^2 = 66.6 \text{ ft}$$

Looking at the data, we notice that times are given to two significant figures, thus h = 67 ft is the most appropriate answer.

TRUE OR FALSE? In equation (7), the term $2h_o$ has a value which is double that of the maximum height of ascent.

[a] CORRECT ANSWER: 1,000 ft

The motion can be divided into three phases. During the acceleration phase ($v_o = x_o = 0$), a distance

$$x_1 = \frac{v_1^2}{2a_1} \tag{1}$$

is covered. Here v is the final velocity of the car during the first phase. It is, of course, also the initial velocity of the car for the second (deceleration) phase. The distance covered during the second phase is given by

$$x_2 = -\frac{v_1^2}{2a_2} \tag{2}$$

The total distance covered, however, is

$$x_1 + x_2 = 3000 \text{ ft} \tag{3}$$

Dividing equation (1) by equation (2) and rearranging terms, we get

$$x_1 = -\frac{a_2}{a_1} x_2 \tag{4}$$

Substituting (4) into (3) yields

$$x_2 = \frac{3000}{1 - (a_2/a_1)} \tag{5}$$

When the values $a_1 = 15$ ft/sec^2 and $a_2 = -30$ ft/sec^2 (positive deceleration $\equiv$ negative acceleration) are substituted into (5), the result $x_2 = 1000$ ft follows.

TRUE OR FALSE? The negative sign preceding the numerical value for a_2 is inserted to indicate that this symbol represents deceleration.

[a] CORRECT ANSWER: 2,400 ft

The motion can be separated into two parts. In the first part, the rocket is subject to a net upward acceleration

$$a_1 = 64 \text{ ft/sec}^2 \tag{1}$$

Taking into account the fact that the initial position and velocity of the rocket are zero, we may write the appropriate equations for this part as follows:

$$h_1 = (1/2)a_1 t_1^2 \tag{2}$$

and

$$v_1 = a_1 t_1 \tag{3}$$

where the y-subscripts have been omitted for neatness.

In the second part of the motion the rocket has initial altitude $h_{02} = h_1$, initial velocity $v_{02} = v_1$, and is subject to the acceleration of gravity ($a_2 = -g$). The final velocity for this part is zero. Thus, the equation involving the velocity and position is written as

$$0 = v_1^2 - 2g(h_2 - h_1); \text{ or } h_2 = h_1 + v_1^2/2g \tag{4}$$

Substituting (2) and (3) into (4) yields

$$h_2 = \frac{1}{2}a_1 t_1^2 + \frac{a_1 2 t_1 2}{2g} = \frac{a_1 t_1^2}{2}\left[1 + \frac{a_1}{g}\right] \tag{5}$$

Using the given values of $t_1 = 5$ sec and $a_1 = 64$ ft/sec^2 in (5), we find

$$h = \frac{64 \times 25}{2}(1 + 2) = 800 \times 3 = 2400 \text{ ft}$$

Let us analyze (5) dimensionally. The dimensions of the expression to the right are

$$(LT^{-2})\ T^2\left[\text{none} + \frac{LT^{-2}}{LT^{-2}}\right] \longrightarrow L$$

This shows that equation (5) is dimensionally correct.

TRUE OR FALSE: In the second part of its motion, the rocket has an initial velocity of zero.

[a] CORRECT ANSWER: 1,200 ft

The motion can be separated into two parts. In the first part, the car is subject to a net acceleration

$$a_1 = 15 \text{ ft/sec}^2$$

Taking into account the fact that the initial position and velocity of the car are zero, we can write the appropriate equations for this part as

$$x_1 = (1/2)a_1 t_1^2$$

and $$v_1 = a_1 t_1$$

In the second part of the motion, the initial position and initial velocities are $x_{02} = x_1$ and $v_{02} = v_1$, respectively, and the car is subject to the deceleration $a_2 = 25 \text{ ft/sec}^2$, and further, the final velocity of the car is zero. Therefore

$$0 = v_{02}^2 - 2a_2 (x_2 - x_{02})$$

or
$$x = x_{02} + [v_{02}/2a_2]$$
$$= x_1 + [v_1^2/2a_2]$$
$$= (1/2)a_1 t_1^2 + a_1^2 t_1^2/2a_2$$

Substitution of the given data yields

$$x = (1/2)\ 15 \text{ ft/sec}^2 (10 \text{ sec})^2 + \frac{(15 \text{ ft/sec}^2)^2\ (10 \text{ sec})^2}{2 \times 25 \text{ ft/sec}^2}$$

$$x = 1200 \text{ ft}$$

TRUE OR FALSE? The magnitude of the acceleration of the car during the second part of its motion is represented by a_2.

[b] CORRECT ANSWER: D

Using the information of the previous question, the problem can be solved by combining this result with $y = y_o + v_{oy} t - gt^2/2$. This, however, involves a quadratic equation. The available information enables us to use a linear equation: $v_y = v_{oy} - gt$. In this equation, v_{oy} and g are known directly, and since $v_y = 0$, then

$$t = (v_{oy} - v_y)/g = 320/32 = 10 \text{ sec}$$

[a] CORRECT ANSWER: True

The maximum height of the projectile is found by using the fact that the y-component of velocity is zero at that point. That is

$$v_y = 0 = v_{oy} - gt_1 \tag{1}$$

where t_1 is the time it takes the projectile to reach its maximum height.

Thus
$$t = \frac{v_{oy}}{g} \tag{2}$$

Then the maximum height is
$$y_{max} = \frac{v_{oy}^2}{g} - \frac{1}{2} g\left[\frac{v_{oy}}{g}\right]^2 = \frac{1}{2}\frac{v_{oy}^2}{g} \tag{3}$$

The range of the projectile is given by the equation

$$R = v_{ox}t_2 \tag{4}$$

where t_2 is the time of flight. Since the time of flight is twice the time that the projectile requires to reach the maximum height,

$$t_2 = 2t_1 \tag{6}$$

Substituting for t_1 from equation (2) into equation (6) we have

$$t_2 = 2v_{oy}/g \tag{7}$$

Hence from (5)
$$R = 2\frac{v_{ox}v_{oy}}{g} \tag{8}$$

And dividing equation (4) by equation (8) we obtain

$$\frac{y_{max}}{R} = \frac{\frac{1}{2}\frac{v_{oy}^2}{g}}{\frac{2v_{ox}v_{oy}}{g}} = \frac{1}{4}\frac{v_{oy}}{v_{ox}} = \frac{1}{4}\tan\theta_0$$

TRUE OR FALSE? As symbolized in this solution, $t_2 = 2t_1$ only if the projectile returns to the same altitude from which it was fired.

[a] CORRECT ANSWER: 69.4 m/sec.

The given quantities are:

Final velocity	$v = 40$ m/sec
Elapsed time	$t = 3$ sec
Acceleration due to gravity	$g = 9.8$ m/sec^2

The sought quantity is:

Initial velocity	$v_o = ?$

The equation used should contain v, v_o, a, and t. Thus, the appropriate one is

$$v = v_o + at = v_o - gt$$

Solving for v_o

$$v_o = v + gt$$

Substitution of the given values

$$v_o = 40 + (9.8 \times 3) = 69.4 \text{ m/sec}$$

[b] CORRECT ANSWER: A

The vertical component of the initial velocity is

$$v_{oy} = v_o \sin\theta = 30 \text{ ft/sec } (1/2) = 15 \text{ ft/sec}$$

When the ball hits the ground, its vertical position will be $y = y_{o_2} = 0$. Using this in the equation $v_y^2 = v_{oy}^2 - 2g(y - y_o)$, we get $v_y^2 = v_{oy}^2$, and $v_y = \pm v_{oy}$. We know that v_y is negative when the ball hits the ground. Since v_{oy} is directed upward, the appropriate sign to use is the negative sign so that $v_y = -v_{oy}$.

It is important to bear in mind when solving problems of this variety that, if the air resistance is negligible, and a projectile is fired at an angle above the horizontal, it returns to its initial altitude with a vertical component of velocity equal and opposite to its initial vertical component. This is the equivalent of saying that its speed at that altitude will be the same as its initial speed since the horizontal component of its velocity remains unchanged.

[a] CORRECT ANSWER: 1.56 ft

Since the total time the ball is in sight is 2 sec, it takes the ball 1 sec to travel the height of the window. Let the speed of the ball at the bottom of the window to be v_o and using the equation

$$y = v_o t - \frac{1}{2} g t^2$$

where y is the height of the window, we find v_o,

$$v_o = \frac{y}{t} + \frac{1}{2} g t$$

Substituting the numerical values, we obtain

$$v_o = \frac{26}{1} \frac{\text{ft}}{\text{sec}} + \left(\frac{1}{2}\right) \left(32 \frac{\text{ft}}{\text{sec}^2}\right) (1 \text{ sec})$$

$$= 42 \text{ ft/sec} \qquad (1)$$

The maximum height from the bottom of the window to which the ball can rise may be obtained from the equation

$$v^2 = v_o^2 - 2gh$$

Noting that at the maximum height, the speed of the ball is zero, we have

$$h = \frac{v_o^2}{2g}$$

Using the value of v_o from (1), we obtain

$$h = \frac{v_o^2}{2g} = \frac{(42)^2}{2(32)} = 27.56 \text{ ft}$$

Thus the height above the window to which the ball can rise is

$$27.56 - 26.00 = 1.56 \text{ ft}$$

TRUE OR FALSE? The highest point the ball can reach measured from the ground is $h = v_o^2/2g$.

[a] CORRECT ANSWER: 20 m/sec

Let v_1 and v_2 be the initial velocity of the first and second projectile, respectively. At the time of collision, these two projectiles will have the same height and the same flight time. Hence we can write,

$$y = v_{1y}t - \frac{1}{2}gt^2 = v_2t - \frac{1}{2}gt^2$$

Solving for v_2 we obtain:

$$v_2 = v_{1y} = v_1 \sin 30° = 40\left[\frac{1}{2}\right] = 20 \text{ m/sec}$$

TRUE OR FALSE? At the time of collision, the two projectiles had been in flight for the same period of time because they had the same initial velocity.

[b] CORRECT ANSWER: 56 ft/sec

The first thing to realize in this problem is that the velocity of the ball when it is at its maximum height is zero. The known quantities then become: $y_o = 0$, $y = 49$ ft, $v = 0$ and $a = -g = -32$ ft/sec^2. The quantity to be computed is the initial velocity v_o.

Looking at the two equations of motion involving the position, namely

$$y = y_o + v_ot + \frac{1}{2}at^2 \qquad \text{and} \qquad v_y^2 = v_o^2 + 2a(y - y_o)$$

we see that the latter is the most appropriate one to use. The former equation involves the elapsed time which has not been given in this problem. It is possible, of course, to compute the time from the equation $v = v_o + at$ and then use it in the first of the above equations. This, however, increases the amount of work we must do, making this method of solution less efficient.

Solving the chosen equation for v_o, with $a = -g$, we get

$$v_o^2 = v^2 + 2g(y - y_o)$$

so

$$v_o = \sqrt{v^2 + 2g(y - y_o)}$$

Substitution gives

$$v_o = \sqrt{0 + (2)(32)(49 - 0)} = \sqrt{(64)(49)} = (8)(7) = 56 \text{ ft/sec}$$

You may check to assure yourself that the units come out right.

[a] CORRECT ANSWER: 0.5 sec

The only acceleration is that due to gravity; namely, $\vec{a} = \vec{g} = 32$ ft/sec^2 in a downward direction. From the equation

$$\vec{v} = \vec{v}_o + \vec{a}t$$

setting $a = -g = -32$ ft/sec^2, $v_{oy} = 0$ and $v_y = -16$ ft/sec, we find

$$t = \frac{-16 \text{ ft/sec}}{-32 \text{ ft/sec}^2} = 0.5 \text{ sec}$$

[b] CORRECT ANSWER: 1.7 m/sec^2

We first look at the known quantities. These are the displacements at various times of travel. From the data, it is seen that $y_o = 0$, $y_1 = -0.708$ m, $y_2 = -1.835$ m, $y_3 = -3.377$ m, and $y_4 = -5.340$ m. The corresponding times are: $t_1 = 0.50$ sec, $t_2 = 1.0$ sec, $t_3 = 1.5$ sec, and $t_4 = 2.0$ sec.

The equation to be used must involve the position and the time. $y = y_o + v_o t + (1/2)at^2$ is such an equation, except that there are two quantities remaining unknown. One is the sought acceleration, and the other is the initial velocity v_o. Since, however, there are more than one pair of y-t data, you can form a set of equations from which you can compute the unknown a.

Since there are two unknowns, we only need two equations. For convenience we may use the data pairs at t = 1 and t = 2. The system becomes

$$-1.835 = (1)(v_o) + \left(\frac{a}{2}\right)(1) \longrightarrow -3.67 = 2v_o + a \qquad (1)$$

and

$$-5.340 = (2)(v_o) + \left(\frac{a}{2}\right)(4) \longrightarrow -5.34 = 2v_o + 2a \qquad (2)$$

Subtracting (2) from (1) we obtain

$$1.67 = -a$$

So,

$$g_m = -a = 1.67 \text{ m/sec}^2$$

Looking at the data, however, you will notice that the times are given to 2 significant figures. Thus, 1.7 m/sec^2 is the most appropriate answer to this question.

[a] CORRECT ANSWER: 20 m

The information directly provided by this problem is the initial velocity of the ball, from which the x- and y-components are easily computed:

$$v_{ox} = v_o \cos\theta \qquad v_{oy} = v_o \sin\theta \qquad (1)$$

The additional information we have is that $a_y = -g = -9.8\ m/sec^2$ and $a_x = 0$. We must find R; i.e., the distance in the x-direction that the ball will travel before hitting the ground. Neglecting air resistance, the horizontal speed remains constant and the equation for the horizontal motion is

$$x = v_{ox}t \qquad (2)$$

In equation (2) we have two unknowns; namely, x and t. However, when x = R, the ball has zero altitude; so y = 0. Use of this additional information along with the equation for the y-direction will help us eliminate t from equation (2). Thus, at y = 0,

$$0 = v_{oy}t - \frac{1}{2}gt^2 \qquad \text{or} \qquad 0 = t(v_{oy} - gt/2)$$

This equation has two roots:

$$t = 0 \qquad \text{and} \qquad t = \frac{2v_{oy}}{g}$$

The first value, t = 0, corresponds to the origin (y = x = 0). Substituting the second value of t into (2) along with x = R, and using (1) we obtain

$$R = v_{ox}\frac{2v_{oy}}{g} = \frac{2(v_o\cos\theta)(v_o\sin\theta)}{g} = \frac{v_o^2(2\sin\theta\cos\theta)}{g}$$

$$= \frac{v_o^2\sin 2\theta}{g} \qquad (3)$$

or

$$R = \frac{(14\ m/sec)^2\sin(2\times 45°)}{9.8\ m/sec^2} = \frac{196}{9.8}\ m = 20\ m$$

[a] CORRECT ANSWER: C

The known quantities, supplied by the question, are:

$t = 1$ sec

$$v_{ox} = v_o \cos\theta = 17.5(0.6) = 10.5 \text{ m/sec} \quad (1)$$

and

$$v_{oy} = v_o \sin\theta = 17.5(0.8) = 14 \text{ m/sec} \quad (2)$$

along with the acceleration of gravity $a = -g = -9.8$ m/sec^2, and $y_o = 0$. Information supplied for the ball released from P is the initial velocity, say u_o, (zero), and the time of fall to the instant of collision.

Obviously, to compute the altitude h_o of P we must use the information that the balls collide, to relate their respective equations of motion. This relationship results from the fact that at the instant the two particles collide, their positions (x- and y-coordinates) are the same. Thus, if h is used to represent the y-coordinate of the ball released from P, at the time of collision we have

$$y \text{ (at } t = 1 \text{ sec)} = h \text{ (at } t = 1 \text{ sec)} \quad (3)$$

The respective equations of motion of the two balls are

$$y = v_{oy}t - \frac{1}{2} gt^2 \quad (4)$$

and

$$h = h_o - \frac{1}{2} gt^2 \quad (5)$$

Making use of equation (3) we get

$$\left[v_{oy}t - \frac{1}{2} gt^2\right]\Big|_{t=1} = \left[h_o - \frac{1}{2} gt^2\right]\Big|_{t=1} \quad (6)$$

Substituting (2) into (6) we find

$$h_o = (14 \text{ m/sec})(1 \text{ sec}) = 14 \text{ m}$$

TRUE OR FALSE? At the time of collision of the two particles they must have the same position coordinates in their common plane of motion.

[a] CORRECT ANSWER: 1.5 mi

At $y = 0$ and $x = R$, we have

$$y = y_o + v_{oy}t + \frac{1}{2} a_y t^2 \longrightarrow 0 = 0 + (v_o \sin\theta)\, t - \frac{1}{2} g t^2$$

or

$$t = \frac{2v_o \sin\theta}{g}$$

Thus,

$$R = v_{ox}t = (v_o \cos\theta)\, t = \frac{v_o^2(2 \sin\theta \cos\theta)}{g} = \frac{v_o^2 \sin 2\theta}{g} \qquad (1)$$

Since in (1) g is constant and v_o is given in the problem, the only parameter we may change, in order to vary R, is the angle θ. Thus, to maximize R, we differentiate R with respect to θ, then set $dR/d\theta = 0$.

From (1)

$$\frac{dR}{d\theta} = \frac{2v_o^2 \cos 2\theta}{g} = 0 \quad \text{or} \quad \cos 2\theta = 0, \text{ and } 2\theta = 90°$$

So,

$$\theta = 45°$$

Thus, 45° is the angle of initial projection for which the maximum range is achieved.

The value of the maximum range becomes

$$R_m = \frac{v_o^2 \sin 90°}{g} = \frac{v_o^2}{g}$$

and for the problem in question

$$R_m = \frac{(500 \text{ ft/sec})^2}{32 \text{ ft/sec}^2} = 7820 \text{ ft} = 1.48 \text{ mi} \approx 1.5 \text{ mi}$$

TRUE OR FALSE? As the angle of departure of a projectile is increased from 70° to 72°, its range increases.

[a] CORRECT ANSWER: 18 ft/sec

Choosing the top of the cliff as the origin we have:

Given quantities: $y_o = 0$; $y = -100$ ft; and $a = -g = -32$ ft/sec

Sought quantity: $v_o = ?$

The most appropriate equation to be used is $y = y_o + v_o t + \frac{1}{2} at^2$.

Solving this equation for v_{oy} we find

$$v_o = \frac{2(y - y_o) - at^2}{2t} \qquad (1)$$

Although this solution involved a simple operation, it is worthwhile demonstrating how a dimensional analysis should be carried out. Since we are looking for the velocity, the dimensions of (1) should be $L/T = LT^{-1}$. The numerator has dimensions

$$[L] - [(LT^{-2})\ T^2] = L$$

The dimension of the denominator is T, giving the correct dimensions, LT^{-1}, for the equation.

When the given values are substituted in (1), we find for the initial velocity

$$v_o = \frac{2(-100 - 0) - (-32)(4)}{2 \times 2} = \frac{-200 + 128}{4} = -18 \text{ ft/sec}$$

The minus sign indicates that the stone was projected downwards. Its initial speed was 18 ft/sec.

[b] CORRECT ANSWER: 32.6 sec

The motion can be divided into two phases; the acceleration and the deceleration phase such that this combination will ensure the required minimum time for the trip.

We note that the final velocity of the acceleration phase is the initial velocity of the deceleration phase, thus

$$v = a_1 t_1 = a_2 t_2 \qquad (1)$$

where t_1 and t_2 are the times necessary to travel the distances in each phase.

next page

continued

The distances covered in each phase are

$$s_1 = \frac{1}{2} a_1 t_1^2$$

$$s_2 = vt_2 - \frac{1}{2} a_2 t_2^2 \qquad (2)$$

Substitution for v from equation (1) leads to

$$s_2 = \frac{1}{2} a_2 t_2^2 \qquad (3)$$

From (2) and (3) we have

$$s = s_1 + s_2 = \frac{1}{2} a_1 t_1^2 + \frac{1}{2} a_2 t_2^2$$

$$= \frac{1}{2} a_1 t_1^2 + \frac{1}{2} \frac{a_1^2}{a_2} t_1^2$$

$$= \frac{1}{2} \frac{a_1}{a_2}(a_1 + a_2)\, t_1^2 \qquad (4)$$

Substituting given numerical values, we get

$$4000 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(10 + 30)\, t_1^2$$

Solving for t_1, we obtain

$$t_1 = 10\sqrt{6} \text{ sec}$$

From (1),

$$t_2 = t_1 \frac{a_1}{a_2} = 10\sqrt{6} \times \frac{1}{3} \text{ sec}$$

This the minimum time necessary to travel the total distance is

$$t_1 + t_2 = 10\sqrt{6}\left(1 + \frac{1}{3}\right) = 32.6 \text{ sec}$$

TRUE OR FALSE? The initial velocity of the deceleration phase is equivalent to the final velocity of the acceleration phase.

[a] CORRECT ANSWER: D

From the given data, the quantity pertaining to horizontal motion is the initial component of the velocity in the x-direction,

$$v_{ox} = v_o \cos 30° = 554 \text{ ft/sec}$$

Since the acceleration due to gravity is in the y-direction, and there is no other acceleration experienced by the shell (with air resistance neglected), v_x will be a constant equal to v_{ox}; i.e.,

$$v_x = 554 \text{ ft/sec}$$

Since v_x is constant throughout the motion, the x-position at any time t is given by

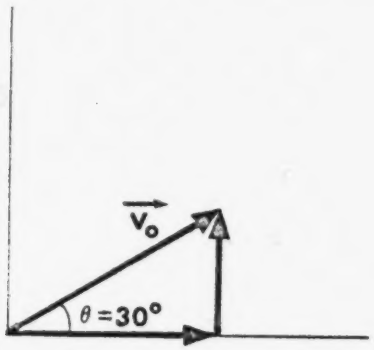
$$x = v_{ox}t$$

The time we are interested in is that found in the solution to the previous problem (page 12[b]), namely, t = 10 sec. Hence,

$$x = 554 \text{ ft/sec} \times 10 \text{ sec} = 5540 \text{ ft}$$

[a] CORRECT ANSWER: A

Let us resolve the initial velocity into its x- and y-components.



$$v_{ox} = v_o \cos 30° = (640)(0.866)$$

$$= 554 \text{ ft/sec}$$

and

$$v_{oy} = v_o \sin 30° = (640)(0.5)$$

$$= 320 \text{ ft/sec}$$

These velocities, along with $y_o = 0$, $a_y = -g = -32$ ft/sec^2, and $a_x = 0$, are the quantities directly given in the problem. At its maximum height the shell has zero vertical velocity; i.e., $v_y = 0$. What we must compute is the maximum height y. Since the elapsed time, until the maximum is reached, is not involved in the question, the equation to be used must be a relation between position and velocity. In one dimension we met a similar situation, and the equation we used was

$$v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$$

The same equation is applicable in two dimensions and may be written as

$$v^2 = v_o^2 + (2\vec{a}) \cdot (\vec{r} - \vec{r}_o)$$

where $\vec{r}$ is the position vector. The x- and y-components of the velocity may be found from the equations

$$v_x^2 = v_{ox}^2 + 2a_x(x - x_o) \quad \text{and} \quad v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$$

Looking at these equations and the given data, we discover that the y-component equation is all we need to solve the present problem. For $v_y = 0$, $y_o = 0$ and $a_y = -g$ the maximum height becomes

$$y_{max} = \frac{v_{oy}^2}{}$$

or

$$y_{max} = \frac{(320 \text{ ft/sec})^2}{64 \text{ ft/sec}^2} = 1600 \text{ ft}$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Newton's First Law of Motion Applied

OBJECTIVE

To analyze and explain the behavior of real bodies at rest or in motion with uniform velocity in terms of Newton's first law of motion.

The first law may be stated in many different ways, and all of them are equally valid. For example, using Newton's phraseology translated directly from the Latin, *Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it.*

If a principle such as this is to be useful in physics, it is not enough to be able to state it or paraphrase it. You must be able to use the significant core of the statement to answer questions and solve problems relating to actual bodies.

A body is in a *state of rest* relative to a given observer if its position with respect to this observer remains fixed over a measurable length of time. If the position changes with time, then the body is in *motion* during that interval. When the body is moving, its velocity is *uniform* if it moves in a straight line with unchanging speed.

The first law continues with the statement that the state of motion of a body may be altered only by impressing a force on it. Since forces are vector quantities, you must recognize that, by an "impressed force" Newton meant an *unbalanced* force. As you already know, two or more forces may be applied to a body so that the resultant force is zero; the forces may be balanced so that no net force exists. In connection with the first law, a zero resultant force is the equivalent of no impressed force at all.

Unbalanced forces that can change the state of motion of a body may stem from gravitational effects, friction, the burning of fuel, and so on. In general, if a body initially at rest is made to start moving, or if a body in motion is caused to change speed or direction, you may conclude that an unbalanced force has acted on it. On the other hand, if two or more forces are applied to a body without succeeding in changing its state of motion, you may be certain that the forces are balanced and that the resultant force is zero.

Before going ahead with this section, you should review the units you might expect to find involved in the first law and its applications. These include the units of mass, force, time, and velocity in both the British and MKS systems.

PROBLEMS

1. A body must be in translational equilibrium if

   A. it is acted upon by a constant force
   B. it has a constant velocity
   C. it has a constant acceleration
   D. no friction forces are involved

2. A particle is set in motion along a frictionless horizontal surface at a speed of one foot per second. What is its speed after ten seconds?

3. A wooden block is set in motion along an infinitely long table. The block slides for several feet and then stops. If the tabletop were made smoother, then one expects that the block would slide farther along the table before stopping. One would conclude that if the tabletop were made so smooth that it offers no resistance to the sliding block, then the block would

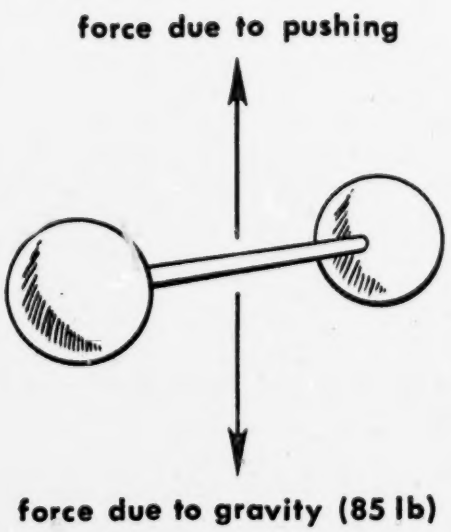
   A. continue to slide forever
   B. accelerate
   C. require an infinite push in order to continue moving indefinitely at uniform velocity
   D. slide farther than in cases where the table is less smooth, but it would still stop

4. A particle is set in motion along a frictionless horizontal surface. After five seconds, the speed of the particle is 3 ft/sec. What was the initial speed of the particle?

5. Please study the figure below.

A weightlifter pushes an 85-lb dumbbell vertically upward at a constant speed of 1 ft/sec. The magnitude of the force he applies to the dumbbell

A. is greater than 85 lb
B. is equal to 85 lb
C. is less as the dumbbell rises
D. is more as the dumbbell rises



INFORMATION PANEL Newton's Second Law of Motion Applied

OBJECTIVE

To analyze and explain the changes in motion of real bodies subjected to unbalanced, externally applied forces in terms of Newton's second law of motion.

The qualitative essence of the second law may be very simply stated like this: an unbalanced force acting on a body brings about a change in its state of motion. A body at rest is caused to start moving; a body already in motion is speeded up, slowed down, or changes its direction of motion when an unbalanced force is impressed on it.

Any of these changes in motion is regarded as *acceleration*. Recall that acceleration is defined as the time rate of change of velocity $\vec{v}$:

$$a_x = dv_x/dt \qquad a_y = dv_y/dt \qquad a_z = dv_z/dt$$

The second law may be stated verbally as follows: the acceleration of a body caused by an unbalanced force varies in magnitude directly as the magnitude of the force and occurs in the same direction as the force, and the acceleration of a body is also inversely proportional to the mass of the body with a given unbalanced force. This may be written mathematically as:

$$\vec{F} = m\vec{a}$$

in which $\vec{F}$ is the force vector, m is the mass, and $\vec{a}$ is the acceleration vector.

next page

continued

Frequently, certain types of situations involving the second law are most easily handled by recognizing that the vector equation above may be written in the form of three scalar equations which relate the x, y, and z components of an unbalanced force to the x, y, and z components of the acceleration for a given mass m. That is,

$$F_x = ma_x \qquad F_y = ma_y \qquad F_z = ma_z$$

Since mass is a scalar quantity, it can be shown that when bodies of mass m_1, m_2, and m_3 (etc.) are rigidly attached to each other to form a single body, they behave physically as a single body of mass ($M = m_1 + m_2 + m_3$).

6. A projectile moves in an x-y plane (horizontal-vertical). The only force on the projectile is the force due to gravity, a force with magnitude w acting vertically downward. The mass of the projectile is m. Which of the following sets of equations (based upon Newton's second law) is correct?

A. $a_x = 0$; $a_y = -w/m$

B. $a_x = -w/m$; $a_y = w/m$

C. $a_x = w/m$; $a_y = -w/m$

D. $a_x = w/m$; $a_y = 0$

NOTE: In the equation $\vec{F} = m\vec{a}$, the units of $\vec{F}$, m, and $\vec{a}$ must be consistent. The usual practice is to specify the units for two of the quantities and to derive the third. The following conventions are in general use:

(i) MKS system - the fundamental mass unit is the kilogram (kg), and the derived force unit is called the newton (nt).

(ii) CGS system - the fundamental mass unit is the gram (gm), and the derived force unit is called the dyne.

(iii) British engineering system - the fundamental unit of force is the pound (lb), and the derived mass is called the slug. This system is sometimes abbreviated by FPS.

These conventions are summarized for your convenience in the following table:

SYSTEM	MASS	FORCE	ACCELERATION
MKS	kilogram (kg)	newton (nt)	meters/second^2 (m/sec^2)
CGS	gram (gm)	dyne	centimeters/second^2 (cm/sec^2)
British (FPS)	slug	pound (lb)	feet/second^2 (ft/sec^2)

7. A net force of 10 lb is applied to a block, causing the block to accelerate at 1 ft/sec^2. What would be the acceleration of the block if the net force were 20 lb?

8. Forces are:

A. scalar quantities
B. vector quantities
C. sometimes scalar, and sometimes vector quantities
D. not scalar quantities and not vector quantities

9. The component equations which are completely equivalent to Newton's second law in three dimensions are:

A. $F_x = ma_x$; $F_y = ma_y$

B. $F\cos\theta = ma_x$; $F = ma_y$; $F\sin\theta = ma_z$

C. $F\cos\theta = ma_x$; $F\sin\theta = ma_y$

D. $F_x = ma_x$; $F_y = ma_y$; $F_z = ma_z$

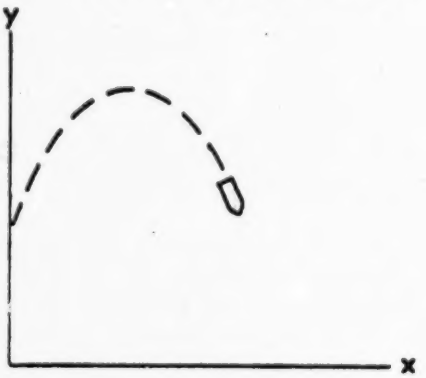
10. A projectile moves in an x-y plane (horizontal-vertical). The only force on the projectile is the force due to gravity, a force with magnitude w acting vertically downward. The mass of the projectile is m. Which of the following sets of equations (based upon Newton's second law) is correct?

A. $a_x = w/m$; $a_y = 0$

B. $a_x = w/m$; $a_y = -w/m$

C. $a_x = 0$; $a_y = -w/m$

D. $a_x = -w/m$; $a_y = w/m$



INFORMATION PANEL — Mass and Weight

OBJECTIVE

To recognize the difference between mass and weight, and to apply this distinction in problems involving these quantities.

In Newton's writings we find mass loosely defined as *the quantity of matter* in a body. Although this is not a rigorous definition, it does serve as a "first bite" into the ideas of dynamics. Everybody is acquainted in a general way with objects of different mass, but to remove ambiguity one distinctly feels the need for a firm and straightforward relationship to establish the meaning of the term.

next page

continued

Consider a specific object to which an unbalanced force is applied to produce acceleration. If the unbalanced force is now doubled, the acceleration of the mass is doubled; if the force is quadrupled, the acceleration is quadrupled. In short, for a given body, the ratio of unbalanced force to the resulting acceleration appears to be constant. That is,

$$\frac{F}{a} = \text{constant}$$

But if you write Newton's second law of Motion as F = ma--we may use the scalar equation if we confine the action to one dimension--it is at once clear that you may also write

$$\frac{F}{a} = m$$

Thus, we may define mass in terms of force and acceleration in this way: the constant ratio of force to acceleration may be considered as an intrinsic property of a body called mass *m*. Mass is, of course, a scalar quantity. Under normal circumstances, the mass of an object does not vary with position, orientation, temperature, pressure, etc. In the MKS system, mass is measured in kilograms and in the British engineering system, in slugs.

The *weight* of a given mass of matter is not a constant, however. Weight is an attractive, gravitational *force* always involving two bodies: the body in question whose weight is to be determined, and a second body such as the Earth or the moon. For a specific body of constant mass, its weight varies as it is moved to different positions with respect to the Earth or when it is carried to some other body such as the moon or Mars.

For a given object, the weight w and mass m are related by

$$w = mg$$

where g is the acceleration due to gravity. This acceleration is approximately 32 ft/sec^2 near the Earth's surface, but it is considerably different near the moon's surface or in outer space. Clearly, the weight varies with proximity to other bodies, even though the mass is constant.

The confusion between mass and weight stems from two causes: (1) there is a popular tendency to express measurements of these quantities in the same units, and (2) at any given position, the weight of an object *is* directly proportional to its mass. To avoid the errors caused by item (1), express mass in *kilograms* or *slugs*, and weight in *newtons* or *pounds*, respectively. Remember that *weight is a force* and that it is a vector quantity, not a scalar like mass.

11. Near the surface of the moon, objects fall with an acceleration of 1.6 m/sec^2. What is the weight of a 3000 gram mass at the moon's surface?

12. Which statement is NOT correct?

A. Mass is an intrinsic property of matter.
B. The mass of a body determines its resistance to a change in motion.
C. Mass is the opposite of inertia.
D. The unit of mass, a kilogram, is a fundamental unit.

13. The weight of a body near the Earth is

A. the attractive force exerted on the body by the Earth
B. the gravitational mass exerted on the body by the Earth
C. not a force, but an intrinsic property of the object
D. a constant force on the object, not subject to variations due to altitude

14. A rock weighs 64 lb on Earth. What does it weigh in free space, and what is its mass in free space? (The unit "slugs" is used as a shorthand notation for $lb\text{-}sec^2/ft$, a unit of mass.)

A. weight in space 64 lb, mass in space 0 slugs
B. weight in space 64 lb, mass in space 2 slugs
C. weight in space 0 lb, mass in space 64 lb
D. weight in space 0 lb, mass in space 2 slugs

15. Near the surface of the moon, objects fall with an acceleration of 1.6 m/sec^2. What is the weight of a 2000 gram mass at the moon's surface?

INFORMATION PANEL — Newton's Third Law of Motion Applied

OBJECTIVE

To use Newton's third law of motion in answering questions and solving problems involving action and reaction forces.

"....If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone; for the rope.....will obstruct the progress of the one as much as it advances that of the other."

In this direct quotation from Newton's writings, you can immediately discern the principal implications of the third law:

A single, isolated force is a physical impossibility.

All forces occur in balanced pairs, the members of the pairs being designated as the action force and the reaction force. This does not mean that there is any difference in their nature, nor does it mean that one force is to be thought of as the "cause" and the other as the "effect."

The two forces do *not* act on the same body; there are always *two* bodies involved. Any force you can describe is merely one aspect of a mutual interaction between two bodies.

In Newton's own words, "To every action there is always opposed an equal and opposite reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

You may possibly find the following statement of greater use to you: *When Body A exerts a force on Body B, then Body B exerts on Body A a force that is equal in magnitude, opposite in direction, and has the same line of action.*

In brief, the two forces resemble one another in that they have the same magnitude and act for the same length of time, but differ in that they act on different bodies and have opposite directions.

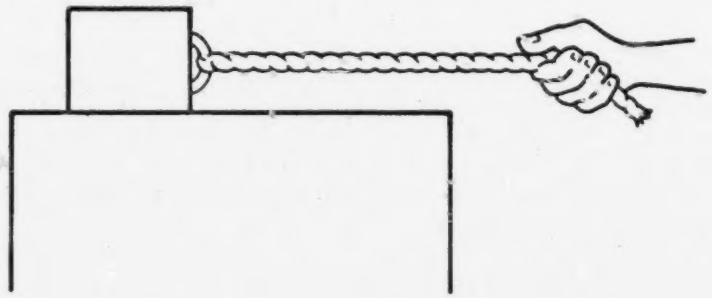
Third law forces are frequently called an action-reaction pair and are related by the expression:

$$\vec{F} = -\vec{F}'$$

Either $\vec{F}$ or $\vec{F}'$ may be considered as the "action" or applied force while the other is the "reaction" force.

16. In the figure below, the reaction force to the force exerted by the hand pulling on the rope is

A. the force of the rope on the block
B. the force of the block on the rope
C. the force exerted by the block on the hand
D. the force exerted by the rope on the hand



17. Choose the most complete correct statement of Newton's third law:

A. If body A exerts a force on body B, then body B exerts an equal and opposite force on body A.

B. If body A exerts a force on body B, then body B exerts a force of equal magnitude on body A.

C. If body A exerts a force on body B, and body B exerts an equal and opposite force on body A, then the forces cancel (the resultant is zero).

D. If body A exerts a force on body B, then body B repels body A with a force of equal magnitude.

18. Bodies A and B exert forces on each other. Either of the two forces may be considered to be

A. due to the motion of A or B
B. perpendicular to the other
C. the reaction force to the other
D. independent of the other

19. Which one of the following statements is true?

A. The attraction of the Earth for an orbiting satellite is an example of a single isolated force.

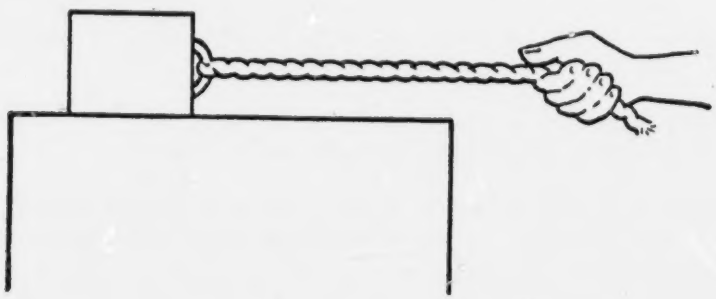
B. Any observable force must be one of a reaction pair involving the interaction of two separate bodies.

C. A force acting on a specific body must be produced by physical contact with another body.

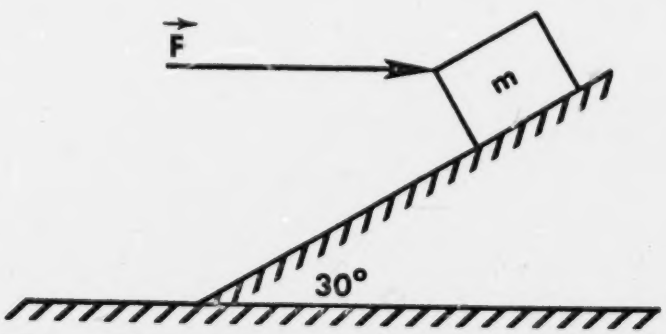
D. When body A exerts a force on body B, the resulting effect on body A cannot be ascertained without a completely separate set of measurements.

20. In the figure below, the reaction force to the force exerted by the rope pulling on the hand is

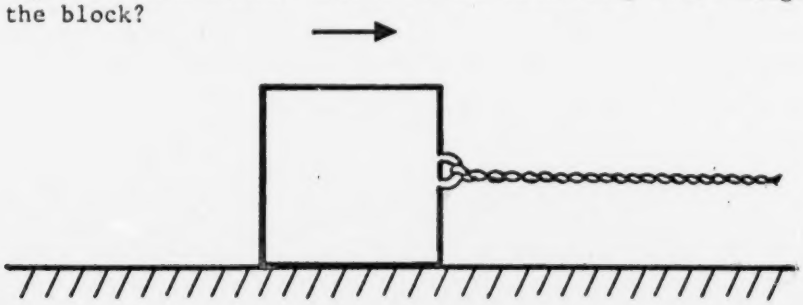
A. the force of the rope on the block
B. the force of the block on the rope
C. the force exerted by the hand on the block
D. the force exerted by the hand on the rope



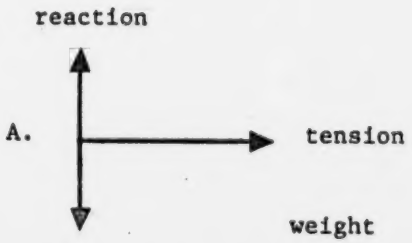
21. A force $\vec{F}$, as shown below, of 10 nt pushes a 3-kg block along a plane inclined at 30°. If $\vec{F}$ is parallel to the horizontal surface, calculate the value of the normal force on the block.



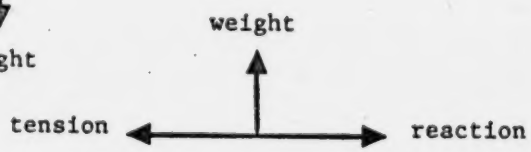
22. A block is pulled along a horizontal, frictionless surface by a horizontal rope as shown below. Which of the following force diagrams applies to the block?



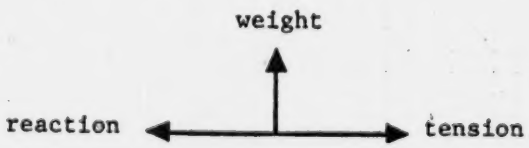
A.



B.



C.



D.

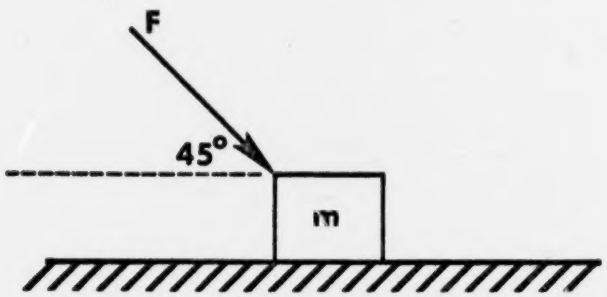
23. Most physics problems in our course result in the formulation of equations of one or more unknowns. We know that the physics of the problem is complete when there are as many independent equations as there are unknowns. Find y for the following set of equations:

$$10x + 20y = 60$$

$$5x - 45y = -80$$

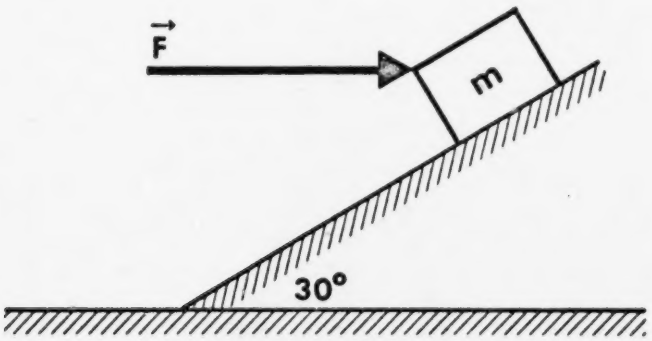
NOTE: The remaining problems in this Segment combine the laws of motion in various ways. Be prepared to apply any or all of the laws in your analysis.

24. A force of 10 nt pushes a 3-kg block along a horizontal frictionless table. What is the value of the force exerted by the table on the block?

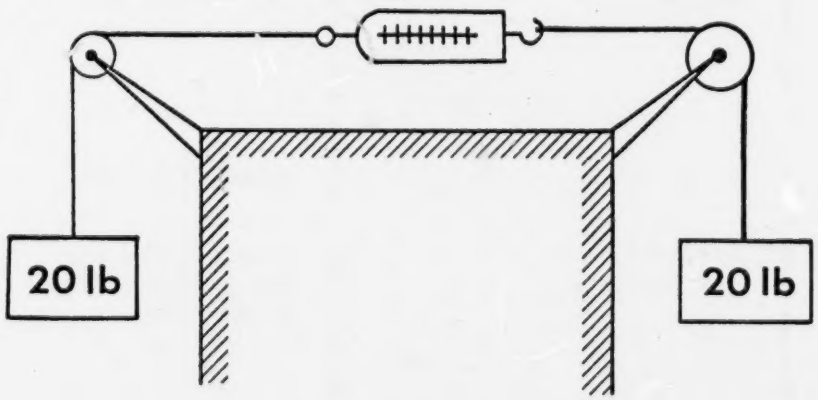


A. 7.1 nt vertically up
B. 36.5 nt vertically up
C. zero
D. 10 nt vertically down

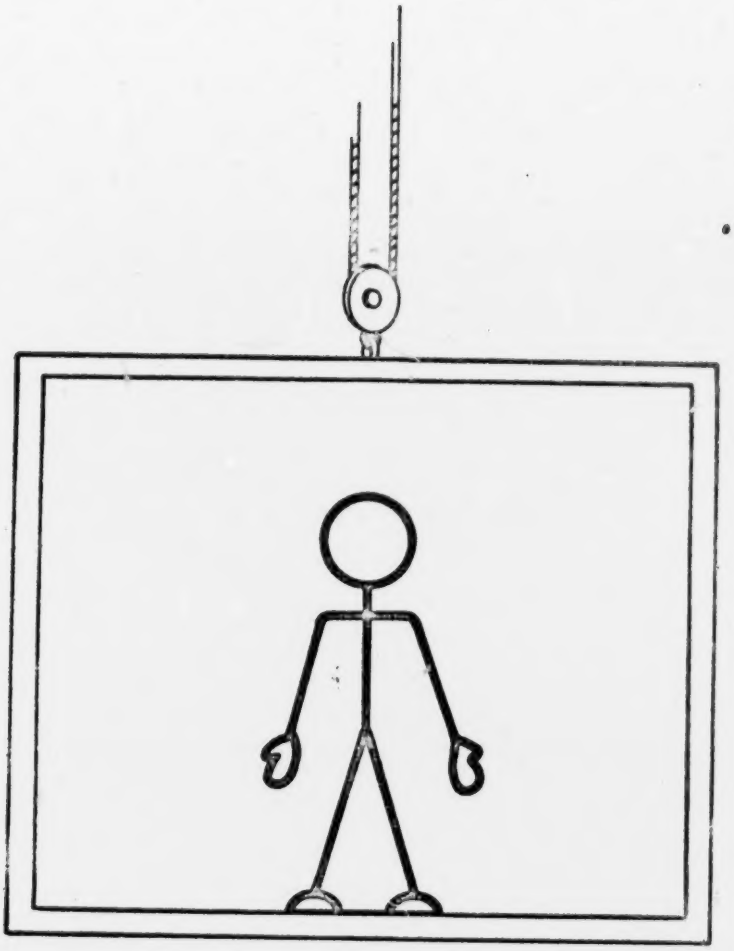
25. A force $\vec{F}$, as shown below, of 20 nt pushes a 3-kg block along a plane inclined at 30°. If $\vec{F}$ is parallel to the horizontal surface, calculate the value of the normal force on the block.



26. Below, two stationary 20-lb blocks are shown attached to a spring balance. The string connecting each block to the balance is massless and the pulleys (different radii) are also massless and frictionless. What is the reading on the spring balance?



27.



In the diagram above, a person stands on the floor of the elevator. The elevator moves upward with uniform speed. Which one of the following free-body diagrams indicates the forces on the person in the elevator?

A

person's weight

B

normal force on the person due to elevator

person's weight

C

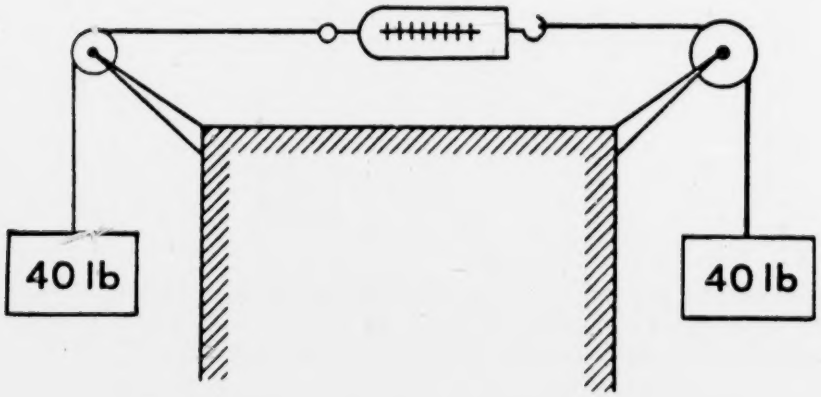
force of the pulley

person's weight

D

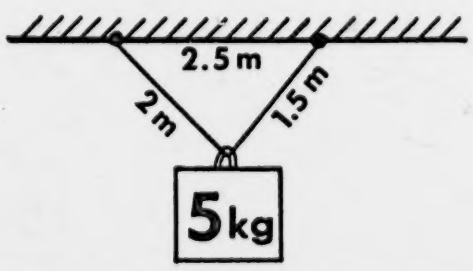
normal force on the person due to elevator

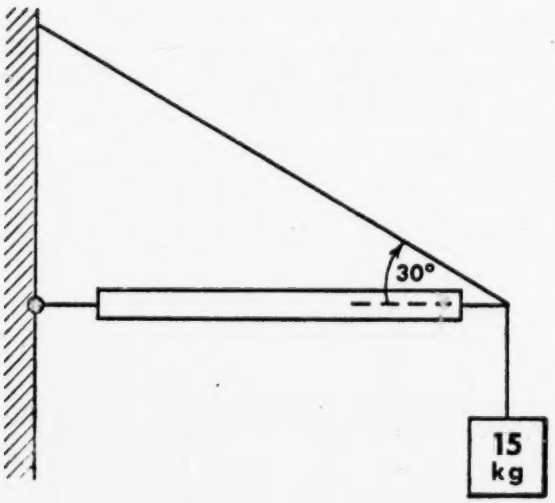
28. Below, two stationary 40-lb blocks are shown attached to a spring balance. The string connecting each block to the balance is massless and the pulleys (different radii) are also massless and frictionless. What is the reading on the spring balance?



29. A mass of 5 kg is suspended from the ceiling by two cords, one of length 1.5 m and the other of length 2 m. The distance between the points of support on the ceiling is 2.5 m. What is the tension in the 2-m cord? (Note: dimensions shown form 3-4-5 triangle.)

A. 3 kg
B. 4 nt
C. 39.2 nt
D. 29.4 nt



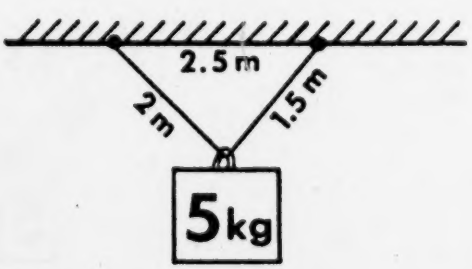


30. In the figure above, a 15-kg mass hangs from a massless beam which is free to rotate at one end. A rope is adjusted in such a manner as to level the beam. What is the tension in the rope?

A. The answer depends upon the shape of the beam.
B. 29.4 nt
C. 30 kg
D. 294 nt

31. A mass of 5 kg is suspended from the ceiling by two cords, one of length 1.5 m and the other of length 2 m. The distance between the points of support on the ceiling is 2.5 m. What is the tension in the 1.5 m cord?

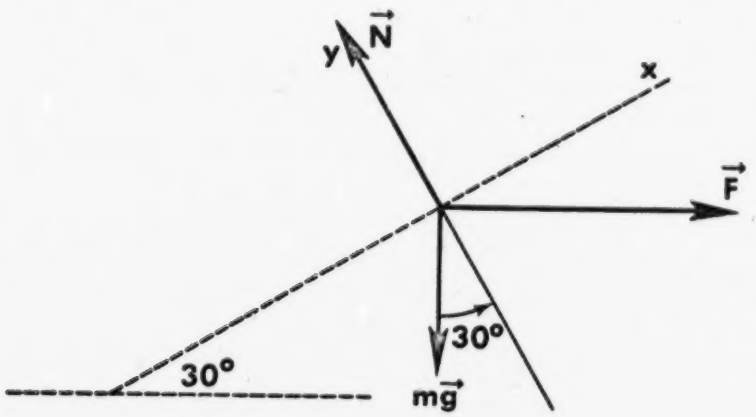
A. 3 kg
B. 4 nt
C. 39.2 nt
D. 29.4 nt



32. A sled of mass m slides down an icy slope that makes an angle θ with the horizontal. Assuming perfectly frictionless conditions, derive general equations for:

(a) the acceleration a of the sled
(b) the resultant (or total) force R acting on the sled
(c) the reaction force N acting on the sled

[a] CORRECT ANSWER: 30.5 nt obtained by the method illustrated below



Notice that we have "tilted" the x- and y-coordinate to accommodate the angle of the plane. This makes life easier and does not change any of our previous sign conventions. The plane can only exert a force on the block *normal* to the plane. The resolution of forces along the x-axis yields,

$$\Sigma F_x = F \cos 30° - mg \sin 30° = ma_x$$

(You can see that we have assumed the block will move up the plane by the fact that the sign of the acceleration is positive.) Along the y-axis,

$$\Sigma F_y = N - F \sin 30° - mg \cos 30° = 0$$

The right-hand side is set to zero because $a_y = 0$ in $\Sigma F_y = ma_y$.

The solution for N (as required) can be obtained from the latter equation.

TRUE OR FALSE? In the diagram above with conditions unchanged, the plane can exert a force on the block at an angle of 60° to the plane.

[a] CORRECT ANSWER: A

A - We will learn in more detail later that two masses attract each other. This is called gravitational attraction. If an object with mass is near the Earth, the Earth's mass attracts the object. This attractive force is equal to the weight of the object. The weight is a force always directed toward the center of the Earth (often, problems refer to this direction as "downward"). The magnitude of the weight can be found from:

$$\text{weight} = w = mg$$

where m is the mass of the object and g is the acceleration of the object when freely falling at the Earth's surface. The attractive force of the Earth for an object decreases as the object moves farther away from the Earth. The weight of a body depends on its position relative to the center of the Earth.

B - INCORRECT - The mass of an object is not the same thing as its weight. It is true that both are measures of an object's resistance to being accelerated, but there is a most important difference. The mass of an object is always the same, regardless of its position in the universe. This is not the case for an object's weight. A 60-lb object on the Earth weighs only 10 lb on the moon and zero lb in outer space. The mass of an object is a measure of the object's inertia or resistance to acceleration, and is a constant for that same object. Weight, however, is a force--the force due to gravitational attraction--and varies with the proximity of other mass objects. Weight at the surface of the Earth, w, is related to mass m by w = mg. This is an important relation and should be memorized.

C - INCORRECT - Weight *is* a force, and arises from the attraction of the Earth for a body. Weight, therefore, depends upon the proximity to the Earth of the body.

D - INCORRECT - The weight of any object is *relatively* constant when it is permitted to remain in the same location relative to a given gravitating mass such as the Earth. But if the object is moved closer to or further from this body as would occur when altitude was changed, the weight would change. Thus, the weight of a body is definitely subject to variations due to altitude.

[a] CORRECT ANSWER: 1 ft/sec

Frictionless means, of course, that there is no frictional force acting to resist the motion of the particle. In the horizontal plane, there are no other net forces present. Therefore, by Newton's first law, an object in motion continues to move in a straight line with constant velocity unless it is acted upon by an externally applied force.

TRUE OR FALSE? If the body were moving to the right and friction were present, the force of friction would act to the left.

[b] CORRECT ANSWER: B

B - Forces have both magnitude and direction; moreover, they obey the laws of addition of vectors. It follows that forces are vector quantities. The force on a particle could be defined as the cause for acceleration. Notice that the direction of acceleration will be the same as the direction of the force.

[c] CORRECT ANSWER: C

C - Mass is a quantitative measure of inertia. The larger the mass of a body, the more difficult it is to alter the body's motion.

A - INCORRECT - By agreement, an intrinsic property of matter is a parameter that does not change with position, time, shape, etc. The mass of a stationary body is independent of physical situations.

B - INCORRECT - A body remaining at rest or moving with a constant velocity continues to exist in this state in the absence of applied forces. A body's resistance to change from such a state is referred to as the body's inertia. Mass is regarded as the quantitative measure of inertia.

next page

continued

D - INCORRECT - In the MKS system of measurement, the kilogram *is* a fundamental unit for mass. You may be confusing the British system and MKS system. In the former, the unit of force is taken as a fundamental unit (that is, the pound) and the unit of mass is derived from it. But in the MKS system, the kilogram is very definitely fundamental.

[a] CORRECT ANSWER: A

A - When the tabletop is made so smooth that it offers no resistance to the sliding block, then there are no forces acting on the block in the direction of motion. Newton's first law states, "Every body persists in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed on it."

B - INCORRECT - When the block is set in motion, your hand exerts a force on the block. The rough plane exerts a force on the block when it slows it down. Notice that both of these forces cause a change in velocity of the block, or one could say the forces produce an acceleration. In the absence of these forces, the block would continue at constant velocity.

C - INCORRECT - When we first start the block moving, we apply a force (push). This push changes the velocity--causes an acceleration--of the block. Therefore, your answer is not completely incorrect because, if the push were continually applied to the block, it would continue to move faster and faster indefinitely.

D - INCORRECT - In order for the block to stop, there must be some resistive force pulling to decrease the velocity of the block.

[a] CORRECT ANSWER: 2 ft/sec^2

In this problem, a given block of unchanging mass is first acted upon by an unbalanced force of 10 lb and then by an unbalanced force of 20 lb. Writing Newton's second law in this form

$$a = \frac{F}{m}$$

it should be clear that, *for a constant mass* as we have in this example the acceleration is directly proportional to the force. Since the force has been doubled, the acceleration must double--going from 1 ft/sec^2 to 2 ft/sec^2.

[b] CORRECT ANSWER: B

B - When a body has a constant velocity it cannot be accelerating. This means that neither its speed nor its direction of motion is changing. Since any unbalanced force acting on the body would cause either its speed, or its direction, or both to change with time, then there can be no unbalanced force impressed upon it. Translational equilibrium is defined as a condition in which the resultant force acting on a body is zero and, as stated in the first law, the body moves with constant velocity.

A - INCORRECT - A body acted on by a constant force must change either its speed or direction in accordance with the first law. However, by definition, translational equilibrium means that the resultant force on the body is zero. A "constant" force and a "zero" force are contradictory.

C - INCORRECT - See the definition of translational equilibrium above. If a body has a constant acceleration, it must be acted on by a constant force (unbalanced) and if this is so, then it cannot be in translational equilibrium.

D - INCORRECT - Friction force is only one kind of many different types of force. The presence of *any* unbalanced force makes translational equilibrium impossible even when friction forces are absent.

next page

continued

TRUE OR FALSE? Speed and velocity are identical physics quantities.

[a] CORRECT ANSWER: A

A - This statement is correct. It implies that a single, isolated force is a physical impossibility, that *two* bodies are always involved when any force acts, and that the action and reaction forces are applied to two different bodies. The other answers violate one or more of these implications. Determine for yourself which one or ones they are.

[b] CORRECT ANSWER: A

A - The forces are given by $F_x = 0$, and $F_y = -w$. Inserting these into $\vec{F} = m\vec{a}$ results in:

$$a_x = 0 \qquad (1)$$

and

$$a_y = -w/m \qquad (2)$$

NOTE: The force which we called w, the force due to gravity, is commonly called the *weight* of the object.

This last result is especially important in establishing the connection between mass and weight near the Earth's surface. Acceleration at the surface of the Earth is given by

$$a_y = -g \qquad (3)$$

where g is approximately equal to 32 ft/sec^2 or 9.8 m/sec^2. From equations (2) and (3) we have:

$$w = mg$$

We will meet this result again. It is the relation between mass and weight at the surface of the Earth.

next page

continued

TRUE OR FALSE? The relationship between mass, weight and gravitational acceleration may also be written as

$$m = g/w$$

[a] CORRECT ANSWER: 4.8 nt

The mass of the moon attracts the mass of the object with a force which we call the weight. This force causes the object to accelerate at 1.6 m/sec^2. Therefore, by Newton's second law,

$$|F| = m|a|$$

$$\text{weight} = (\text{mass})(\text{moon's acceleration of gravity})$$

$$w = (3 \text{ kg})(1.6 \text{ m/sec}^2) = 4.8 \text{ nt}$$

TRUE OR FALSE? The weight of a stone on Mars is the force of gravitational attraction between the stone and Mars.

[b] CORRECT ANSWER: D

D - Now you might anticipate that many problems involving forces and acceleration are best studied by resolving the forces into their x, y, and z components. Newton's second law of motion is written as $\vec{F} = m\vec{a}$. In three dimensions the vector $\vec{F}$ has three components F_x, F_y, and F_z. The vector on the right-hand side $\vec{a}$ is multiplied by a scalar m. Therefore, the components of the vector on the right-hand side are ma_x, ma_y, and ma_z. From the properties of vectors, we can write the following scalar equations:

$$F_x = ma_x$$

$$F_y = ma_y$$

$$F_z = ma_z$$

In this manner, three separate one-dimensional problems result; that is, motion in the x-direction, motion in the y-direction, and motion in the z-direction.

next page

continued

A - INCORRECT - Although it is often sufficient just to consider the motion of a particle in an x-y plane, Newton's second law applies, in general, to motion in three dimensions.

B - INCORRECT - The component form of $\vec{F} = m\vec{a}$ may be found by writing $\vec{F}$ and $\vec{a}$ in component form:

$$\vec{F} = F_x + F_y + F_z$$

$$\vec{a} = a_x + a_y + a_z$$

C - INCORRECT - You should be thinking of the components of the force in all three dimensions.

[a] CORRECT ANSWER: C

C - In dealing with "third law" forces, either one many always be considered to be the action force and the other the reaction force.

[b] The procedure used in the solution of this problem is identical with that presented for problem 11. For your convenience, the solution is reproduced below with appropriate numerical substitutions.

CORRECT ANSWER: 3.2 nt

The mass of the moon attracts the mass of the object with a force which we call the weight. This force causes the object to accelerate at 1.6 m/sec^2. Therefore, by Newton's second law,

$$|F| = m|a|$$

$$\text{weight} = (\text{mass})(\text{moon's acceleration of gravity})$$

$$w = (2 \text{ kg})(1.6 \text{ m/sec}^2) = 3.2 \text{ nt}$$

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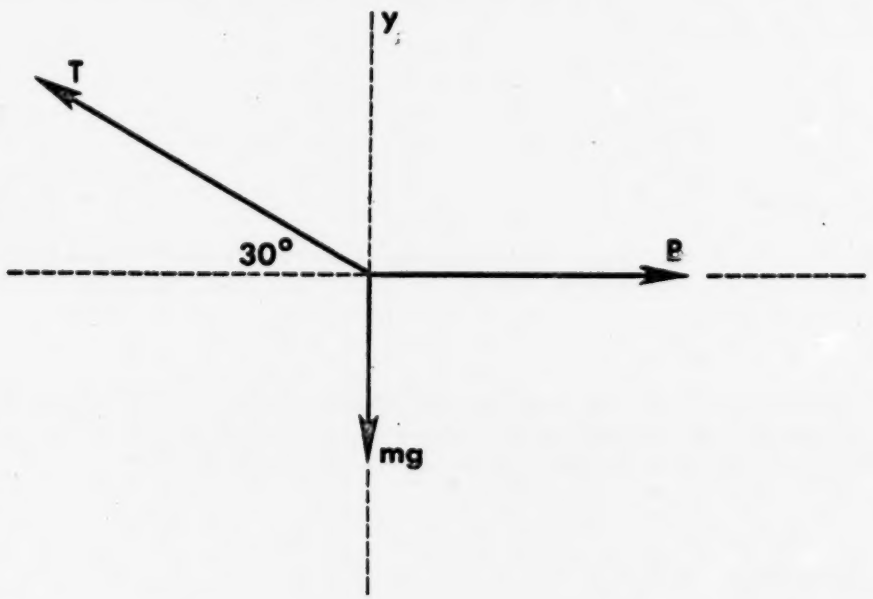
TRUE OR FALSE? If a large meteor is very distant from any other mass, its weight may be quite small.

[a] CORRECT ANSWER: D

D - A force diagram can help to solve the problem.



There is no motion in this problem, thus,

$$\Sigma F_y = T \sin 30° - mg = 0$$

$$\Sigma F_x = B - T \cos 30° = 0$$

and therefore,

$$T = 2mg$$

$$T = 294 \text{ nt}$$

[a] CORRECT ANSWER: B

B - Since the dumbbell moves at constant velocity in the y-direction, the sum of all external forces on the dumbbell in the y-direction must be zero. A force of 85 lb (the weight) pulls down. Therefore, the weightlifter must push up with a force equal to 85 lb.

Mathematically, the fact that the sum of the forces in the y-direction must be zero is written as

$$\Sigma F_y = 0$$

$$\underset{\substack{\text{force due}\\ \text{to pushing}}}{f} + \underset{\substack{\text{weight, the minus sign indicates that the weight acts downward}}}{(-w)} = 0$$

Therefore,

$$f = w$$

$$= 85 \text{ lb}$$

TRUE OR FALSE? If the dumbbell had weighed 60 lb, the weightlifter would have had to exert 60 lb upward to keep it moving upward at constant speed.

A, C, D - INCORRECT - Recognize that the dumbbell moves at constant speed and that, according to the first law of motion, the total force acting upon it is zero.

Approach the question again and consider the forces on the dumbbell. They are the force due to gravity and the force due to pushing. The sum must be zero.

[b] CORRECT ANSWER: B

B - This is an implication of Newton's third law; forces never occur singly--they always occur in pairs of equal magnitude acting in opposite directions on each of the two bodies involved in the interaction.

next page

continued

A - INCORRECT - The attraction of the Earth for the satellite may be referred to as the "action." The satellite, however, attracts the Earth with an equal and oppositely directed force. This is the reaction force. Hence, there is a pair of forces to be considered.

C - INCORRECT - The Earth-satellite example shows why this statement is not true. Gravitation (among other forces) acts without physical contact between bodies.

D - INCORRECT - A separate set of measurements is not required. If you know the magnitude and direction of the force acting on body B, you immediately know the magnitude and direction of the force on body A by applying the third law.

[a] The procedure used in the solution of this problem is identical with that presented for problem 26. For your convenience, the solution is reproduced below with appropriate numerical substitutions.

CORRECT ANSWER: 40 lb

The system is in equilibrium. This tells you that there is no unbalanced force acting on either of the two blocks. The tension in the string must equal the weight of either block so that the net force in the vertical direction is zero. The string has no mass so that the tension in the string must be the same throughout. Finally, the spring balance provides a direct measurement of tension.

Consider this in a slightly different light: suppose the 40-lb weight attached to the ring on the left side of the balance were replaced by a rigid wall. You would then be quite confident that the balance would read 40 lb, the wall providing the reaction force to the tension in the string. Now consider the left-hand 40-lb weight as providing the same reaction force and you will at once see that the reading of the balance will not change. It will still read 40 lb.

TRUE OR FALSE? If the system were not in equilibrium, there would be an unbalanced force acting on each of the two blocks.

[a] CORRECT ANSWER: D

D - Whenever two bodies interact, the force exerted by the first body on the second is equal in magnitude and opposite in direction to the force exerted by the second body on the first.

A, B, C - INCORRECT - We have previously agreed that Newton's third law can be simply stated as follows: "If body A exerts a force on body B, then body B exerts an equal and opposite force on body A." Now replace body A by the hand and body B by the rope and you will see the results clearly.

TRUE OR FALSE? If body A pushes eastward on body B with a force of 6 nt, then body B pushes eastward on body A with a force of 6 nt.

[b] CORRECT ANSWER: B

B - Notice that in your free-body diagram the force up (normal force) is equal to the force down (weight). This makes sense for the following reasons:

(1) The velocity was given as constant and upward.

(2) For constant velocity, the sum of all external forces on a body must be zero.

A - INCORRECT - The force of attraction of the Earth (gravity) acts on the mass of the passenger to give the weight force down. However, the person moves upward at constant speed (zero acceleration). Newton's first law tells us that in the absence of exterior forces an object will continue at constant velocity. There must, therefore, be another force. What is it?

C - INCORRECT - When you are preparing a free-body diagram, you should be very careful to include only those forces that are actually acting on the object. In the figure, the force of the pulley acts on the elevator.

next page

continued

D - INCORRECT - Two forces act on the passenger. The force of the Earth's attraction (gravity) pulls vertically downward, and the elevator exerts a force normal to the floor of the elevator (upward). Note also that the elevator moves up at constant speed.

[a] The procedure used in the solution of this problem is identical with that presented for problem 6. For your convenience, the solution is reproduced below.

CORRECT ANSWER: C

C - The forces are given by $F_x = 0$, and $F_y = -w$. Inserting these into $\vec{F} = m\vec{a}$ results in:

$$a_x = 0 \qquad (1)$$

and

$$a_y = -w/m \qquad (2)$$

NOTE: The force which we called w, the force due to gravity, is commonly called the *weight* of the object.

This last result is especially important in establishing the connection between mass and weight near the Earth's surface. Acceleration at the surface of the Earth is given by

$$a_y = -g \qquad (3)$$

where g is approximately equal to 32 ft/sec^2 or 9.8 m/sec^2. From equations (2) and (3) we have:

$$w = mg$$

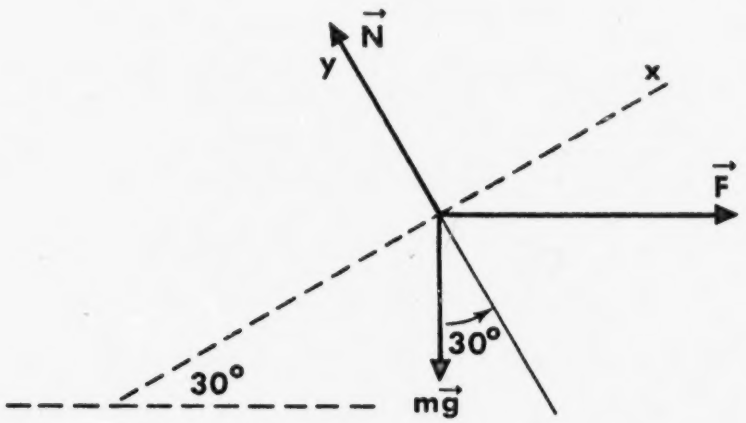
We will meet this result again. It is the relation between mass and weight at the surface of the Earth.

TRUE OR FALSE? The relationship between mass, weight and gravitational attraction may also be written as

$$m = w/g$$

[a] The procedure used in the solution of this problem is identical with that presented for problem 21. For your convenience, the solution is reproduced below with appropriate numerical substitutions.

CORRECT ANSWER: 35.5 nt obtained by the method illustrated below



Notice that we have "tilted" the x- and y-coordinate to accommodate the angle of the plane. This makes life easier and does not change any of our previous sign conventions. The plane can only exert a force on the block *normal* to the plane. The resolution of forces along the x-axis yields,

$$\Sigma F_x = F \cos 30° - mg \sin 30° = ma_x$$

(You can see that we have assumed the block will move up the plane by the fact that the sign of the acceleration is positive.) Along the y-axis,

$$\Sigma F_y = N - F \sin 30° - mg \cos 30° = 0$$

The right-hand side is set to zero because $a_y = 0$ in $\Sigma F_y = ma_y$.

The solution for N (as required) can be obtained from the latter equation.

TRUE OR FALSE? The plane can only exert a force on the block *normal* to the plane because the plane is considered frictionless.

[a] CORRECT ANSWER: D

D - The weight of an object near the Earth is a force resulting from the attracting of the mass of the Earth and the mass of the object. At the Earth's surface, the weight is

weight = (mass of object)(acceleration due to gravity)

$$|w| = |mg|$$

The direction of the force is downward.

When the object is in free space, it is no longer attracted by the Earth or any other mass. Then, of course, the weight is zero.

The mass is an intrinsic property of the rock. Its value does not change. The value of the mass may be found from

$$m = w/g = 64 \text{ lb}/(32 \text{ ft/sec}^2)$$

$$m = 2 \text{ slugs}$$

A - INCORRECT - We define free space as that area in outer space where the gravitational attraction of the Earth and other astronomical bodies is zero for a particular object. So, in this problem the weight is zero. Mass is a property intrinsic in the rock and is not affected by the rock's location, hence cannot be zero in this problem.

B - INCORRECT - See the statement of A above. The mass is given correctly here, but the weight is zero for the reasons given.

C - INCORRECT - The weight is given correctly in this answer, but the unit used for mass is incorrect. In the British system, mass is measured in slugs, not pounds.

[a] CORRECT ANSWER: $y = 2$

The original equations: $10x + 20y = 60$ (1)

$5x - 45y = -80$ (2)

Multiply equation (2) by 2 and then subtract it from (1).

$$\begin{array}{r} 10x + 20y = 60 \\ -10x + 90y = 160 \\ \hline 110y = 220 \\ y = 2 \end{array}$$

[b] CORRECT ANSWER: A

A - A free-body diagram shows all the forces acting *on* an object and is very helpful in solving problems. As you have noted:

(1) The weight as a force is always directed vertically downward and equal to mg.

(2) The reaction is a force perpendicular to the surface and directed vertically upward. This force is often called the "normal" force.

[c] The procedure used in the solution of this problem is identical with that presented for problem 2. For your convenience, the solution is reproduced below.

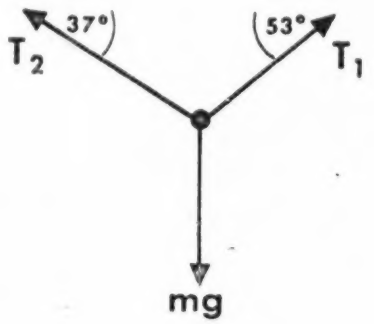
CORRECT ANSWER: 3 ft/sec

Frictionless means, of course, that there is no frictional force acting to resist the motion of the particle. In the horizontal plane, there are no other net forces present. Therefore, by Newton's first law, an object in motion continues to move in a straight line with constant velocity unless it is acted upon by an externally applied force.

TRUE OR FALSE? As a sled moves horizontally on the snow, the friction force on its runners acts in the direction the sled moves.

[a] CORRECT ANSWER: D

D - This static problem should bring to mind the use of Newton's first law. You probably recognized that the triangle is a 3-4-5 triangle with angles approximately 37°, 53°, and 90°. A free-body diagram is, of course, a must.



$$\Sigma F_y = T_1 \sin 53° + T_2 \sin 37° - mg = 0$$

$$\Sigma F_x = T_1 \cos 53° - T_2 \cos 37° = 0$$

We now have two independent equations and two unknowns, and thus a solution is possible.

TRUE OR FALSE? If the shortest side of a 3-4-5 triangle is 6 cm long, then the longest side is 8 cm long.

[b] CORRECT ANSWER: 20 lb

The system is in equilibrium. This tells you that there is no unbalanced force acting on either of the two blocks. The tension in the string must equal the weight of either block so that the net force in the vertical direction is zero. The string has no mass so that the tension in the string must be the same throughout. Finally, the spring balance provides a direct measurement of tension.

Consider this in a slightly different light: suppose the 20-lb weight attached to the ring on the left side of the balance were replaced by a rigid wall. You would then be quite confident that the balance would read 20 lb, the wall providing the reaction force to the tension in the string. Now consider the left-hand 20-lb weight as providing the same reaction force and you will at once see that the reading of the balance will not change. It will still read 20 lb.

next page

continued

TRUE OR FALSE? The tension on the string in this problem is considered the same throughout because the string is massless.

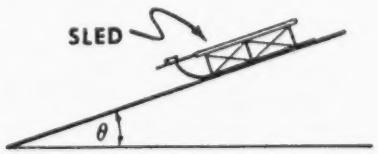
[a] CORRECT ANSWERS: Indicated below

(a) $\Sigma F_x = ma_x$ but since $\Sigma F_x = -mg \sin\theta$,

$-mg \sin\theta = ma_x$, hence $a_x = -g \sin\theta$

(b) Since $a_x = -g \sin\theta$ then the resultant force

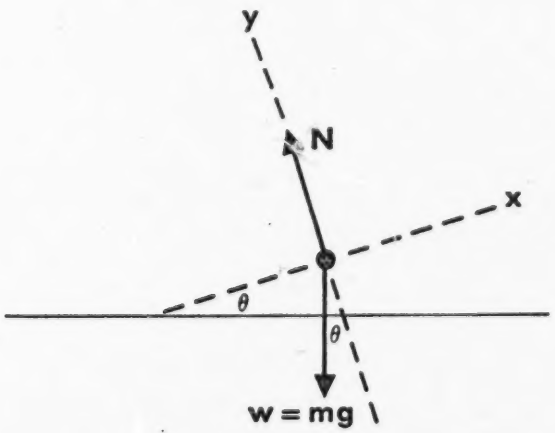
$$\vec{R} = -mg \sin\theta \, \hat{i}$$



(c) $\Sigma F_y = 0$

$N = mg \cos\theta = 0$

so $N = mg \cos\theta$



TRUE OR FALSE? In this solution, the force identified by N is the force exerted by the plane on the sled at right angles to the plane.

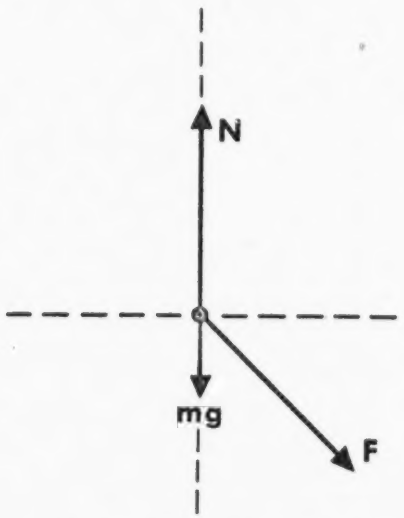
[a] CORRECT ANSWER: B

B - The table can produce a force only in a direction perpendicular to its surface at the point of contact with the block. This is the "normal" force N.

Hence, $\Sigma F_y = N - mg - F\cos 45°$

$N = F\cos 45° + mg$

$N = 36.5$ nt (vertically upward)



FREE-BODY DIAGRAM

[b] The procedure used in the solution of this problem is identical with that presented for problem 16. For your convenience, the solution is reproduced below with appropriate alterations.

CORRECT ANSWER: D

D - Whenever two bodies interact, the force exerted by the first body on the second is equal in magnitude and opposite in direction to the force exerted by the second body on the first.

A, B, C - INCORRECT - We have previously agreed that Newton's third law can be simply stated as follows: "If body A exerts a force on body B, then body B exerts an equal and opposite force on body A." Now replace body A by the rope and body B by the hand and you will see the results clearly.

next page

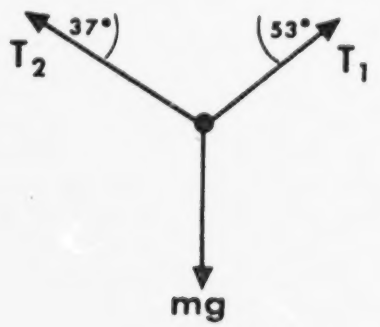
continued

TRUE OR FALSE? A body may move with uniform velocity only if the resultant force acting on it is zero.

[a] The procedure used in the solution of this problem is identical with that presented for problem 29. For your convenience, the solution is reproduced below with appropriate alterations.

CORRECT ANSWER: C

C - This static problem should bring to mind the use of Newton's first law. You probably recognized that the triangle is a 3-4-5 triangle with angles approximately 37°, 53°, and 90°. A free-body diagram is, of course, a must.



$$\Sigma F_y = T_1 \sin 53° + T_2 \sin 37° - mg = 0$$

$$\Sigma F_x = T_1 \cos 53° - T_2 \cos 37° = 0$$

We now have two independent equations and two unknowns, and thus a solution is possible.

TRUE OR FALSE? In this diagram, the vector mg represents the resultant of T_1 and T_2.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Newton's Laws of Motion Combined

OBJECTIVE

To solve problems involving the laws of motion in various combinations.

The problems contained in this section call for the application of Newton's three laws of motion for a number of different systems. The following problems are based on the principles of these laws. The third law, involving pairs of forces, is usually applicable to any problem in dynamics. As you begin work on these problems, be prepared to use all of the laws of motion in combination.

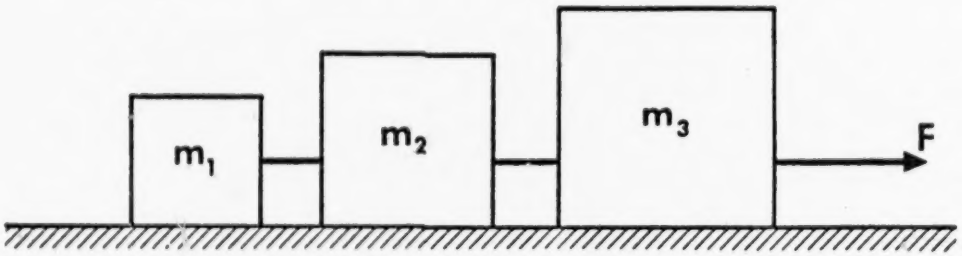
The following points are worth emphasizing:

> The tension in a "massless" or unaccelerated continuous string is the same in all parts of the string and is equal to the reading that a spring balance would give if inserted anywhere along the length of the string.
>
> A free-body diagram isolates the body under consideration from other parts of the system and indicates the magnitudes and directions of all the forces acting on it. Care must be taken to maintain a consistent set of sign conventions for the force directions.

In this section you will be expected to (a) draw free-body diagrams accurately so that they may be used to help you solve relevant problems; (b) determine tensions in strings; (c) find the acceleration of a different system.

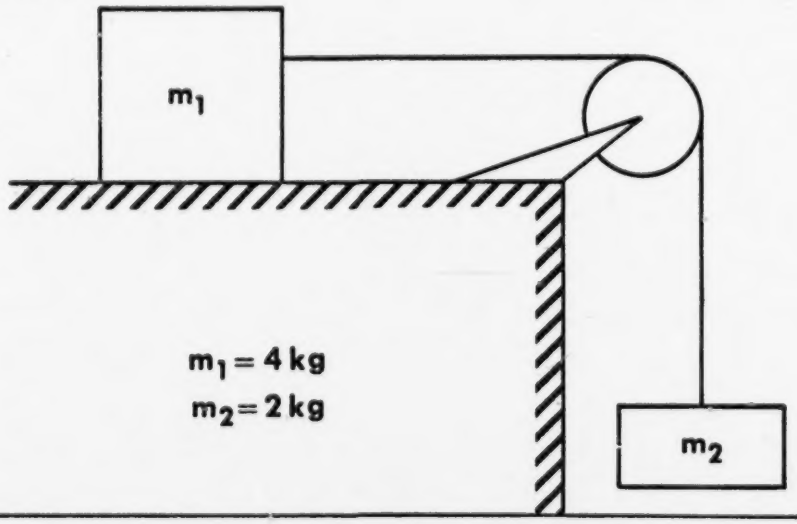
PROBLEMS

1. In the figure below, a force F of 90 nt accelerates three blocks of mass m_1 = 20 kg, m_2 = 30 kg, and m_3 = 40 kg. What is the tension in the cord connecting m_1 and m_2? (The plane is frictionless.)

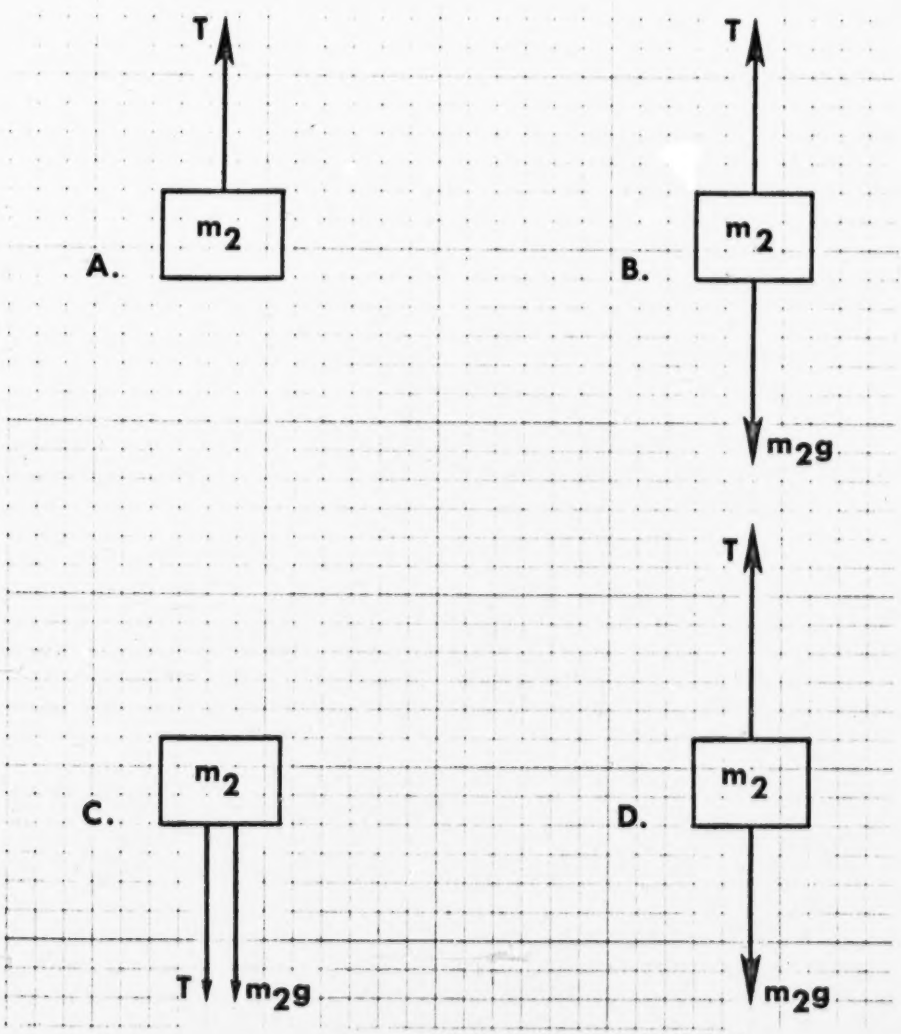


A. 20 nt
B. 40 nt
C. 90 nt
D. 30 nt

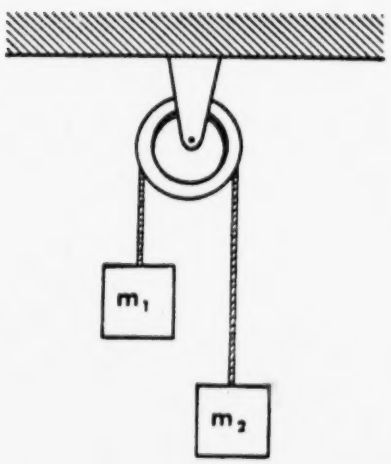
2. Referring to the figure, what is the acceleration of m_1? Assume the idealized conditions of a frictionless table and massless pulley and cord. Be sure to draw a free-body diagram and to use a consistent sign convention.



3. In the figure of the previous problem, a block of mass m_1 on a smooth horizontal surface is pulled by a string which is attached to a block of mass m_2 hanging over a pulley. The string and pulley are assumed to be massless and frictionless, and a smooth surface means we are also to ignore friction on that surface. Choose a correct free-body diagram for the block of mass m_2 showing all the forces acting upon it. Use the symbol T for the tension (magnitude) in the cord.



4. Two unequal masses m_1 = 5 kg and m_2 = 10 kg are connected by a string which passes over a pulley as shown in the figure. Assume the idealized conditions of a massless, frictionless pulley and string. Find the acceleration of the masses.



INFORMATION PANEL

The Coefficients of Friction

OBJECTIVE

To define and use the coefficients of static and kinetic friction in answering questions and solving problems in which frictional forces must be considered.

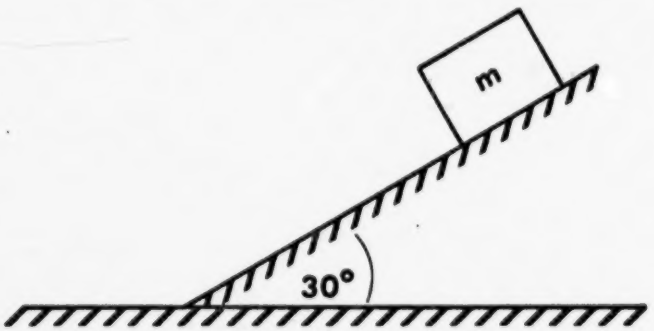
An orange crate rests on a rough wooden plank. If you drag the crate along the plank, you can readily feel the retarding force of friction. If the force you apply is too small to move the crate, a frictional force still exists, but is balanced by the applied force. Thus, frictional forces may act even when there is no relative motion between the surfaces of contact.

If no horizontal force acts on the crate, the force of friction is zero; as the horizontal force increases, the retarding force also grows proportionately until the instant that motion begins. Motion begins when the magnitude of the external force becomes larger than the maximum frictional force.

Once motion begins, the magnitude of the frictional force drops below the value it had just before motion started. This reduced force is known as *kinetic friction* while the force acting to hold the crate stationary before it begins to move is called *static friction*.

In the work that follows, you will be asked to (a) predict what a specific object would do while being acted upon by external and frictional forces; (b) determine the values of coefficients of friction between definite surfaces under given conditions of rest of motion; (c) calculate the changes of motion of objects which are subject to several forces, including friction.

5. In the diagram, you can see a 30 lb block on an inclined plane with coefficient of kinetic friction $\mu_k = 0.40$ and coefficient of static friction $\mu_s = 0.60$. What will the block do when released from rest?



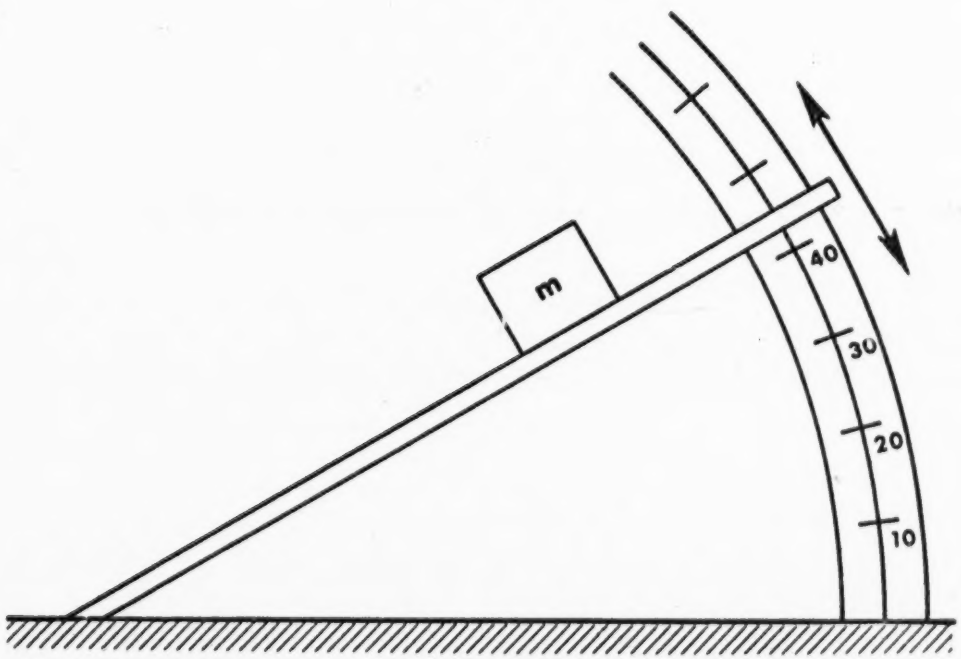
A. remain at rest
B. slide with constant velocity down the plane
C. accelerate down the plane

6. The force of kinetic friction

A. is proportional to the applied force
B. is always greater than that of static friction
C. is proportional to the normal force on the surface of contact
D. depends upon the relative speeds of the surfaces in contact

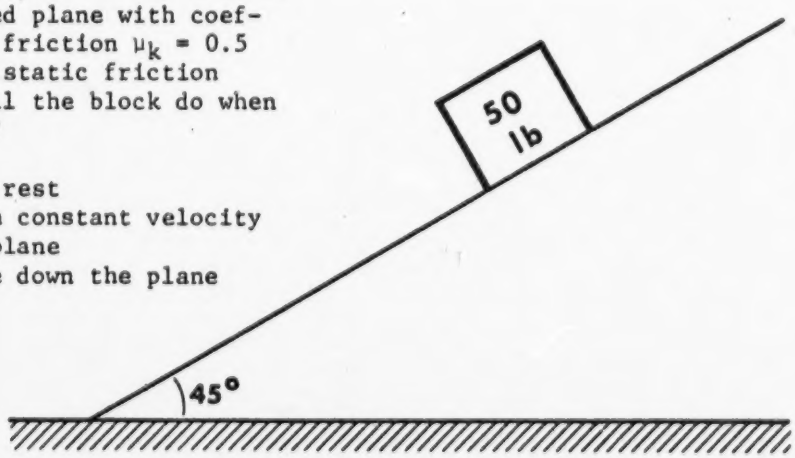
7. The force of kinetic friction always acts in a direction *opposite* that of the motion.

8. A stationary 20-lb block is placed on an inclined plane of variable angle. The angle of the incline is slowly increased until, finally, at 41 degrees the block begins to move. What is the coefficient of static friction of this surface?



9. In the diagram, you can see a 50 lb block on an inclined plane with coefficient of kinetic friction $\mu_k = 0.5$ and coefficient of static friction $\mu_s = 0.8$. What will the block do when released from rest?

A. remain at rest
B. slide with constant velocity down the plane
C. accelerate down the plane



INFORMATION PANEL Friction on an Inclined Plane

OBJECTIVE

To solve problems in which the force of friction is one of the forces which causes changes in the motion of a body on an inclined plane.

One method of measuring the coefficient of static friction is to place the object in question on the surface and then tilt the surface at greater and greater angles until the object just begins to slide. The angle at which motion is just about to begin under these conditions is known as the *angle of repose*. It can be shown that the tangent of the angle of repose is equal to the coefficient of static friction for the two surfaces or

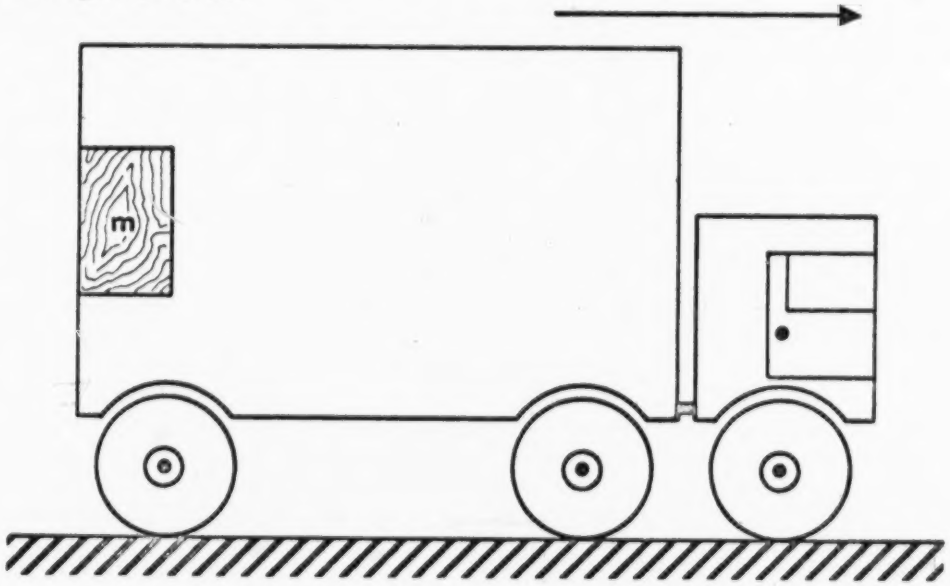
$$\mu_s = \tan\theta$$

where μ_s is the coefficient of static friction and θ is the angle of repose. The derivation of this relationship is presented in your textbooks and also in one of the problem solutions in this section. When the angle of incline is smaller than the angle of repose, the object remains stationary; when it is larger, the object slides downward with approximately uniform acceleration. To find the coefficient of kinetic friction, then, one would determine the angle at which the body slides down with uniform *speed* and apply the same relationship but with the static coefficient replaced by the kinetic coefficient.

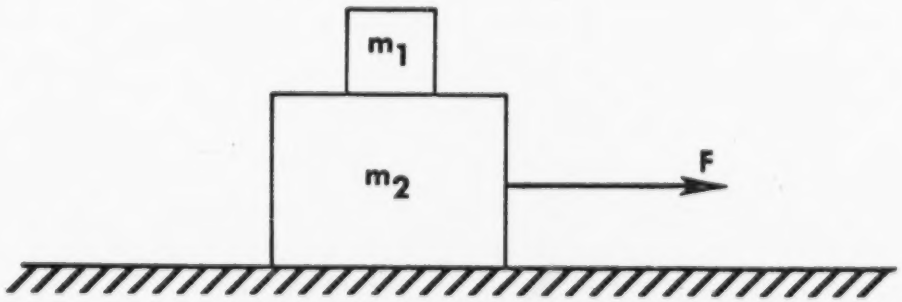
It is anticipated that you will be able to (a) calculate both coefficients of friction when given sufficient data; and (b) apply the idea of angle of repose to simplify the solutions of applicable problems.

10. Refer to the diagram in problem 5. The block slides down the plane inclined at 30° with constant velocity. What is the coefficient of kinetic friction?

11. The truck shown in the diagram accelerates at 30 m/sec^2 with a 20-kg block stuck to the back door as a result of the friction between it and the door. The coefficients of static and kinetic friction are 0.5 and 0.5 respectively for all surfaces inside the truck. If the truck begins to reduce its acceleration, at what acceleration will the block begin to fall?



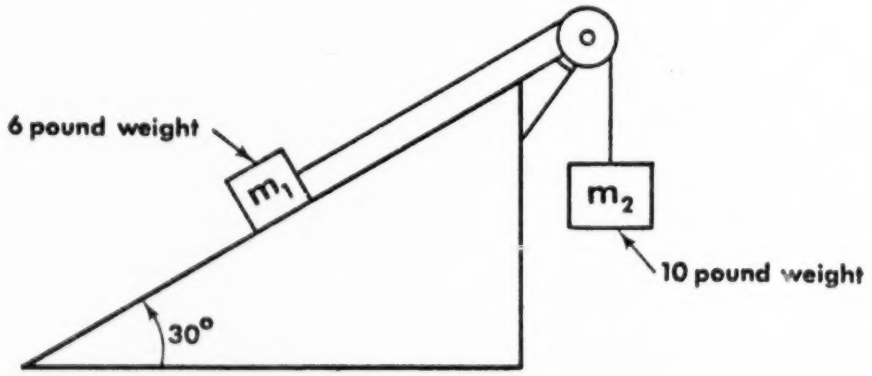
12.



In the diagram, a force F of 136 nt pulls two blocks along a horizontal surface (m_1 = 10 kg, m_2 = 20 kg). The coefficients of static and kinetic friction for all surfaces are μ_s = 0.055 and μ_k = 0.040.

True or false? Block one will *move to the left* relative to block two.

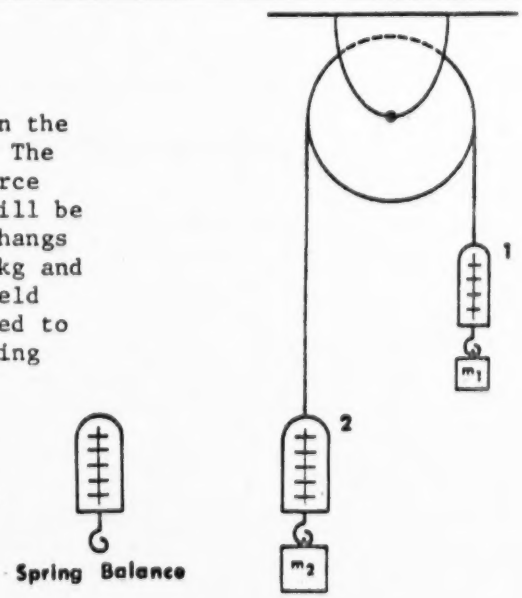
13. As shown in the figure, a 6-pound weight on the inclined plane (coefficient of kinetic friction between the weight and the surface is $\mu_k = 0.2$) is connected by a light inextensible string to a 10-lb weight. The string passes over an ideal frictionless, massless pulley. What is the magnitude and direction of the acceleration of the 10-lb weight?



A. 8.7 ft/sec^2 upward
B. 15. ft/sec^2 upward
C. 16. ft/sec^2 downward
D. 7.4 ft/sec^2 downward

14. The pulley and the string shown in the figure are assumed to have zero mass. The spring balances are used to measure force and for the purposes of this problem will be assumed to have zero mass. One block hangs from each spring balance with $m_1 = 10$ kg and $m_2 = 20$ kg. At first, the pulley is held stationary and the string is not allowed to slip. What is the reading of each spring balance?

A. $F_1 = 10$ nt, $F_2 = 20$ nt
B. $F_1 = 98$ nt, $F_2 = 196$ nt
C. $F_1 = 130.6$ nt, $F_2 = 130.6$ nt
D. $F_1 = 9.8$ nt, $F_2 = 19.6$ nt



15. The pulley in the figure of the previous question is released allowing blocks 1 and 2 to move. What are the scale readings on the spring balances?

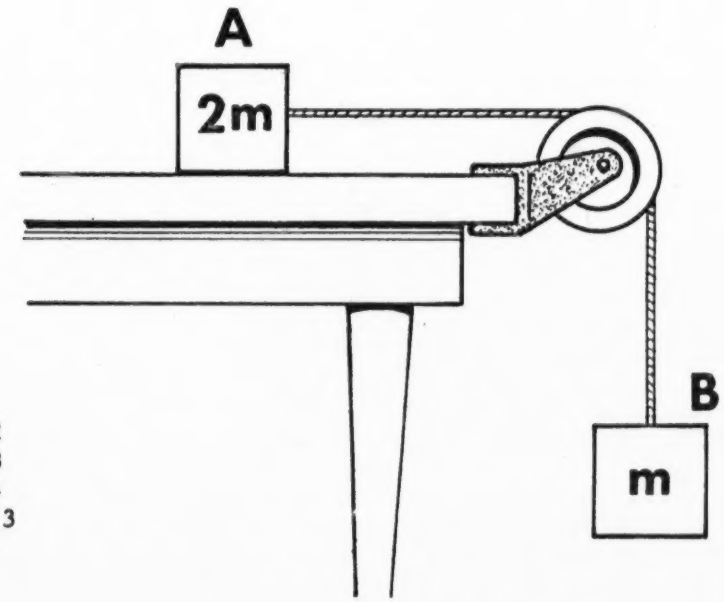
A. $F_1 = 294$ nt
   $F_2 = 294$ nt

B. $F_1 = 130.6$ nt
   $F_2 = 130.6$ nt

C. $F_1 = 98$ nt
   $F_2 = 196$ nt

D. $F_1 = 10$ nt
   $F_2 = 20$ nt

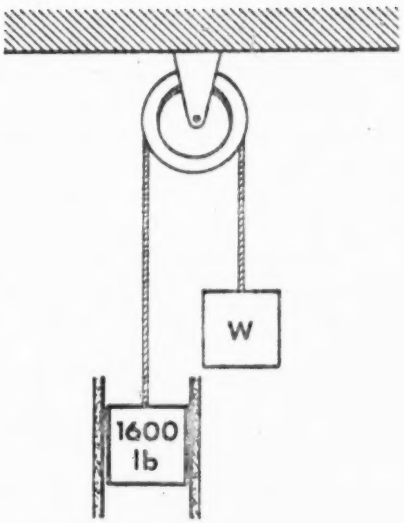
16. In the figure below, the pulley and cord are assumed to be weightless, and the table is assumed to be a frictionless surface. Blocks A and B have masses 2m and m, respectively; find an expression for the acceleration (magnitude) of the blocks.



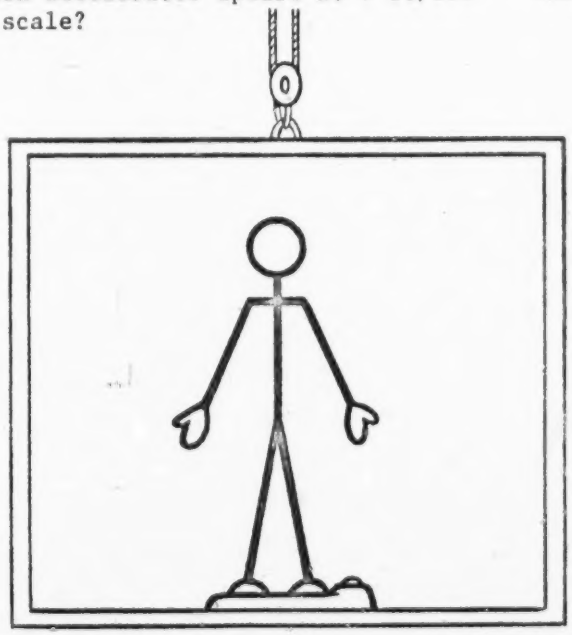
A. g/2
B. g/3
C. g/4
D. 2g/3

17. What weight w must be hung from the right-hand side of the cable in order to give the 1600-lb elevator cage shown in the figure an upward acceleration of 8 ft/sec^2. The friction force between the cage and the girders opposes the motion with a total force of 100 lb. Consider the pulley massless and frictionless. The cable is massless.

A. 1500 lb
B. 2800 lb
C. 3500 lb
D. 4000 lb



18. In the figure below, a man is standing in an elevator which is initially stationary. The weight scale under the man reads 160 lb. The elevator then accelerates upward at 5 ft/sec^2. What is the new reading on the scale?



19. The elevator of the previous question accelerates until it reaches a velocity of 20 ft/sec at which time it continues at a constant velocity. The scale under the man now reads what weight?

20. The cable of the elevator in the figure of problem 18 suddenly breaks and the elevator begins to fall. As the elevator falls down the shaft, what weight does the scale read?

21. A man is standing on a scale in a sealed elevator. What is the direction and the minimum magnitude in MKS units of the acceleration of the elevator when the scale reads zero?

[a] CORRECT ANSWER: C

C - The force of kinetic friction experienced by an object is approximately proportional to the normal force N applied to that object. In symbols,

$$f_k \propto N$$

or

$$f_k = \mu_k N$$

The constant of proportionality μ_k is called the coefficient of kinetic friction. Similar statements may be made for the force due to static friction f_s.

[b] CORRECT ANSWER: 160 lb

From Newton's first law we know that if an object is stationary or moving at constant velocity, the sum of all external forces must be zero. In this case that means the scale must push up with a force equal to the weight of the man (or 160 lb).

[c] CORRECT ANSWER: B

B - In problems of this type it is often best to assume a direction of motion and allow the signs to indicate the correct direction. For example, let's assume m_2 goes up and m_1 down. After drawing a free-body diagram for each mass, one should have the equations:

$$T - m_2 g = m_2 a$$

and

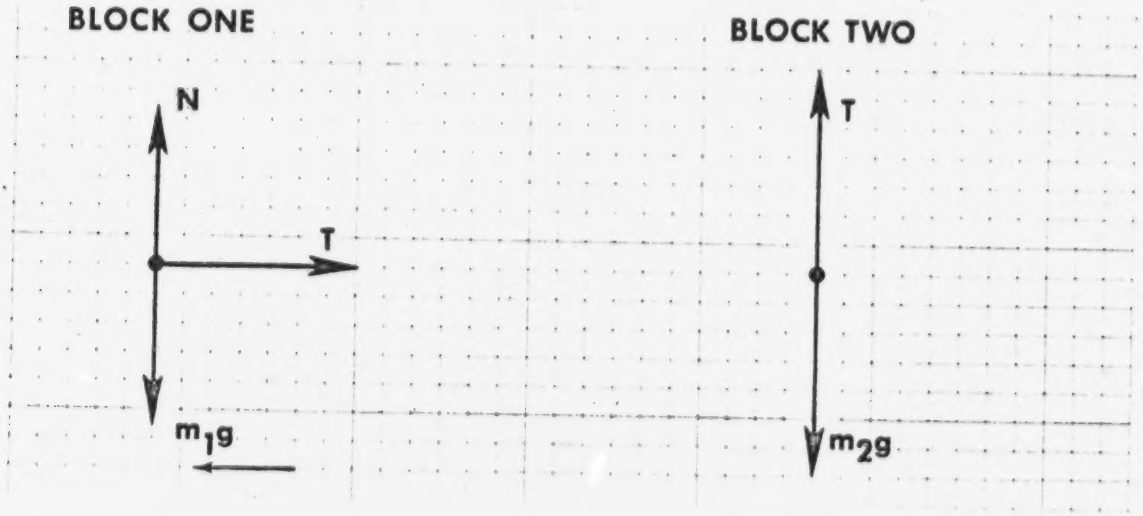
$$T - m_1 g = m_1 (-a)$$

Notice that the accelerations of the masses are equal in magnitude, but they differ in signs. If the numerical value of the acceleration a is negative, our assumption was wrong and m_2 goes down and m_1 up (this is the case in our problem).

[a] CORRECT ANSWER: 3.26 m/sec^2

Free-body diagrams are very useful in physics since they isolate the forces on a body. In this problem, we have two bodies of interest and thus two free-body diagrams.

Three forces act on block one; weight m_1g, the normal force N of the table against the block, and the tension T in the cord. Only weight m_2g and tension T act on block two.



Block One	Block Two
$\Sigma F_x = T = m_1a_1$	$\Sigma F_x = 0$
$\Sigma F_y = N - m_1g = 0$	$\Sigma F_y = T - m_2g = -m_2a_2$

Since the masses are attached by the cord, the magnitude of the accelerations a_1 and a_2 are equal: $a_1 = a_2$

Notice that we have taken the following sign convention:

(a) up and right are positive

(b) down and left are negative

next page

continued

Solving two simultaneous equations

$$T = m_1 a_1$$

$$T - m_2 g = -m_2 a_1$$

for a_1 gives

$$a_1 = m_2 g/(m_1 + m_2)$$

TRUE OR FALSE? The sign conventions used in this solution are: right (+), left (-), down (+), up (-).

[a] CORRECT ANSWER: TRUE

If block one adheres to block two, then the acceleration of both blocks due to the force F can be found from Newton's second law,

$$F - \mu_k(m_1 + m_2)g = (m_1 + m_2)a$$

$$a = 4.14 \text{ m/sec}^2$$

In order for block one to accelerate at 4.14 m/sec^2, the maximum static friction force must be large enough to insure this acceleration. Therefore, the question is:

$$\mu_s m_1 g \geq m_1 a \text{ ?} \quad \text{(where } m_1 g \text{ is the normal force)}$$

The static friction-force is not large enough in this problem and block one will move to the left relative to block two.

TRUE OR FALSE? The coefficient of kinetic friction μ_k must be known if this problem is to be solved successfully.

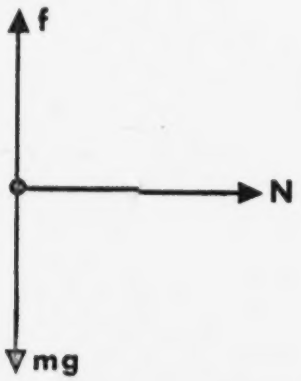
[b] CORRECT ANSWER: B

B - Only two forces act on block two. The tension T in the string and the force due to gravity $m_2 g$.

[a] CORRECT ANSWER: 19.6 m/sec^2

In this problem we know the frictional forces suspend the block in the vertical direction. We also know that, in order to stay with the truck, the block must have the same acceleration as the truck. The normal force is available to produce this acceleration of the block.

A free-body diagram is shown below.



Applying Newton's second law in the vertical and horizontal directions gives

$$\Sigma F_y = f - mg = 0 \qquad \text{(if the block sticks)}$$

$$\Sigma F_x = N = ma$$

Therefore, we find that

$$\mu_s N = mg$$

$$N = ma$$

Solving for the critical acceleration below which the block would begin to slide, we obtain,

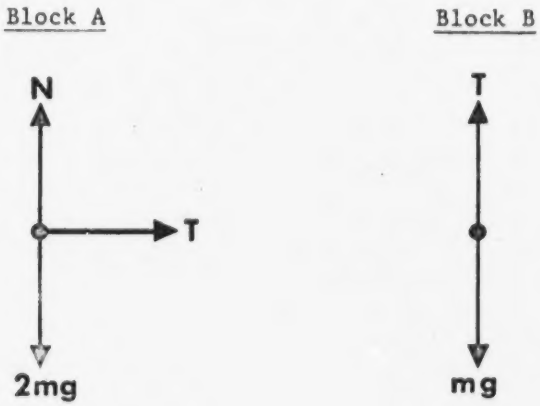
$$a = \frac{g}{\mu_s}$$

Notice once again that the solution of this problem is independent of the mass of the objects concerned.

TRUE OR FALSE? If you substitute the known numbers for μ_s and g in $a = g/\mu_s$, you get 39.2 m/sec^2 as the answer.

[a] CORRECT ANSWER: B

B - Let N be the normal force of the table on block A and let T be the magnitude of the tension in the cord. The free-body diagrams for both blocks are shown below.



Applying Newton's second law, we have

Block A

$$\Sigma F_x = T = (2m)a$$

$$\Sigma F_y = N - 2mg = 0$$

Block B

$$\Sigma F_y = T - mg = m(-a)$$

Solving for a, we obtain

$$a = g/3$$

[b] CORRECT ANSWER: TRUE

Friction tends to retard the motion of an object. Consequently, the frictional force on a moving body always acts in a direction opposite to the direction of motion.

[a] CORRECT ANSWER: A

A - Here is a problem where we can first treat the blocks as a group to find the acceleration, then we isolate only block one to calculate the tension. Since there is no friction, the sum of the forces in the x-direction yields,

$$\Sigma F_x = F = (m_1 + m_2 + m_3)a$$

Therefore,

$$a = 1.0 \text{ m/sec}^2$$

Now for block one to accelerate at 1.0 m/sec^2, a force must be applied. The only force available in this problem is the tension in the cord connecting block one and block two.

Therefore,

$$\Sigma F_x = T = m_1 a$$

$$T = 20 \text{ nt}$$

TRUE OR FALSE? The force required to accelerate Block 1 is the tension in the cord connecting Block 2 and Block 3.

[b] CORRECT ANSWER: 9.8 m/sec^2 downwards

The sum of forces in the y-direction acting on the mass yields

$$N - mg = ma$$

where N is the force of the scale on the person. We are told that this force N = zero. Therefore,

$$-mg = ma$$

or

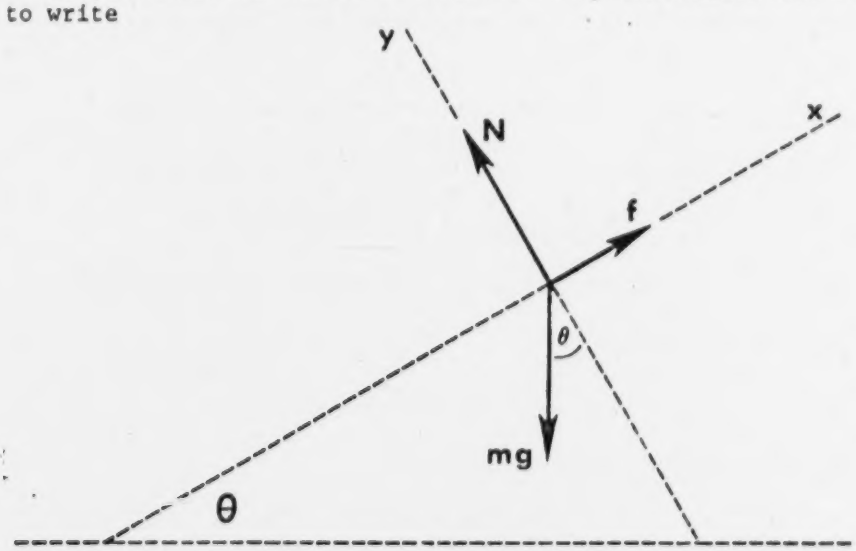
$$a = -g = -9.8 \text{ m/sec}$$

The magnitude of the acceleration of the elevator is 9.8 m/sec and the direction is downwards.

TRUE OR FALSE? The reading of the weight scale in this problem is independent of the mass of the man.

[a] CORRECT ANSWER: 0.578

This problem yields a useful method for determining the coefficient of kinetic friction. If one has a plane whose angle of inclination can be varied, an angle can be chosen for which the sliding velocity is constant. Then the forces in the x- and y-directions can be summed up to write



$$\Sigma F_y = N - mg\cos\theta = 0$$

$$\Sigma F_x = \mu_k N - mg\sin\theta = 0$$

Taking the ratio

$$\frac{\mu_k N}{N} = \frac{mg\sin\theta}{mg\cos\theta}$$

yields

$$\tan\theta = \mu_k$$

TRUE OR FALSE? In the diagram, the force of friction is symbolized by N.

[a] CORRECT ANSWER: A

Refer to the free-body diagram. There you will notice that a stationary object on a frictional surface will resist the applied force with an equal and opposite force until the block begins to move. After motion has begun, the frictional force remains constant and equal to $\mu_k N$, where μ_k is the coefficient of kinetic friction and N is the normal force. In other words, for motion to begin,

$$\mu_s N < \text{applied force along x-axis}$$

In our problem the sum of the forces in the y-direction gives

$$\Sigma F_y = N - mg \cos 30° = 0$$

and the projection of the weight along the x-axis is $mg \sin 30°$.

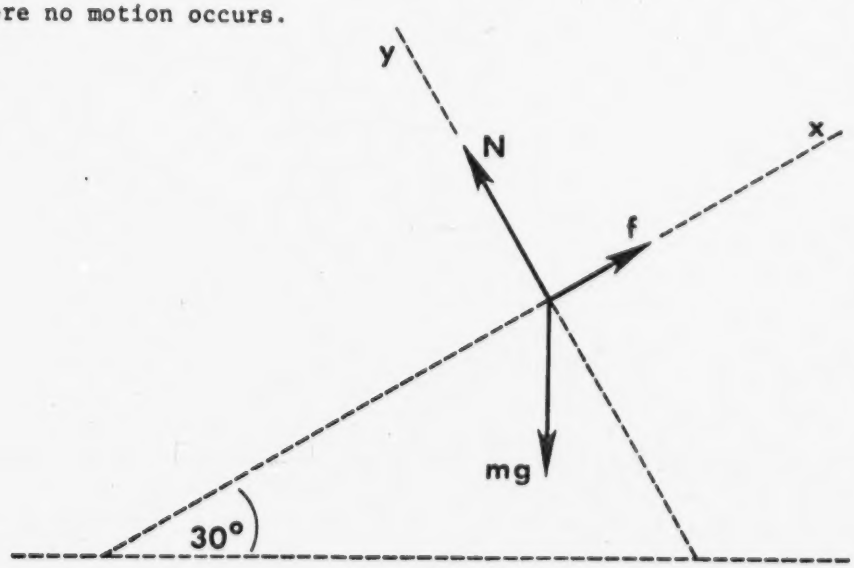
We must test the following: is

$$\mu_s N < mg \sin 30° \ ?$$

In this problem you have found that

$$\mu_s N > mg \sin 30°$$

Therefore no motion occurs.



TRUE OR FALSE? In this problem, the frictional force varies in direct proportion to the displacement.

[a] CORRECT ANSWER: 3.27 m/sec^2

The free-body diagrams for masses m_1 and m_2 are shown below.

Block of mass m_1 Block of mass m_2

Since the two blocks are connected by the same string, the tension is the same in both diagrams.

mass m_1	mass m_2
$\Sigma F_x = 0$	$\Sigma F_x = 0$
$\Sigma F_y = T - m_1 g = m_1 a$	$\Sigma F_y = T - m_2 g = -m_2 a$

Note that the magnitude of the acceleration is the same for both masses and since $m_2 > m_1$, m_2 has acceleration in the negative y-direction and m_1 has acceleration in the positive y-direction.

Solving the simultaneous equations for a one obtains:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = 3.27 \text{ m/sec}^2$$

TRUE OR FALSE? The sign conventions used in this solution are: up (+), down (-), right (+), left (-).

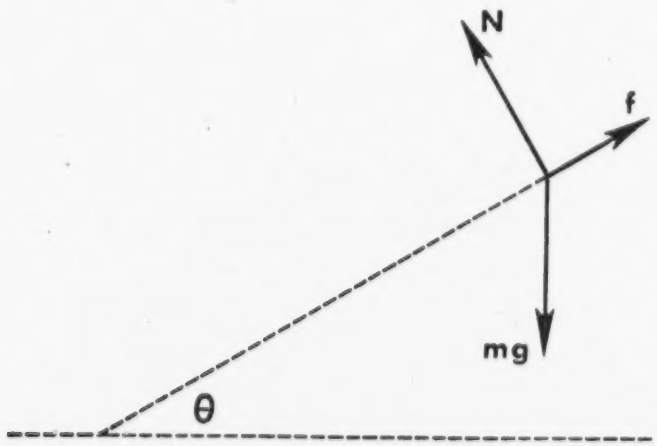
[b] CORRECT ANSWER: B

B - The spring balances measure the actual force transmitted by the string. If the masses are stationary, the string must exert a force upward equal to the weight of each block. The pulley is not free to move and thus the tension in the string is not equal throughout the length of the string.

[a] CORRECT ANSWER: 0.869

Your reasoning may be something like this. If the block is going to move at all, the planar component of the weight must be slightly greater than the retarding frictional force.

The free-body diagram appears below.



If the applied force is slightly greater than the force at the peak, the block will move.

One may write the force equation in the following form:

$$\mu_s N = mg \sin\theta$$

and

$$N = mg \cos\theta$$

where θ is the angle at which the block starts to move.

Thus

$$\mu_s = \tan\theta$$

Inserting $\theta = 41°$ completes the problem.

[a] CORRECT ANSWER: C

C - It is clear from the diagram below that if

$$\mu_s N < \text{applied force}$$

the body begins to move. The sum of forces in the y-direction gives

$$\Sigma F_y = N - mg \cos 45° = 0$$

and the projection of weight in the x-axis is

$$mg \sin 45° = \text{applied force}$$

Therefore, $\mu_s N = \mu_s mg \cos 45° = (0.8)(50)(0.707)$ lb

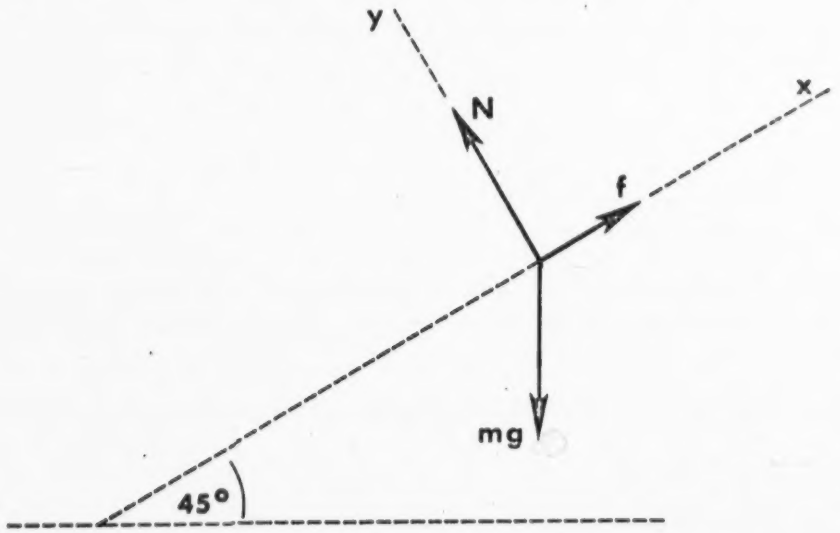
and applied force = $mg \sin 45° = (50)(0.707)$ lb

Hence, motion will begin. Once motion begins we should use the coefficient of kinetic friction to find whether the body moves with constant velocity or is being accelerated.

For the body to accelerate

$$\mu_k N < \text{applied force}$$

Since $\mu_k < \mu_s$, the body will accelerate down the plane.



TRUE OR FALSE? To determine whether or not motion occurs, we must observe if the force of friction is larger or smaller than the component of weight along the plane.

[a] CORRECT ANSWER: C

C - In many cases similar to this problem we really don't know initially whether block two (10 lb) will move up or down. It is better to assume a direction and then let the signs indicate the correct direction. Stick with your assumption of the direction of motion throughout the problem. Notice that the x-direction is parallel to the plane and the y-direction is perpendicular to the plane.

For example, let's assume that block one (6 lb) moves down the plane. Then the equations of motion would be

Block One

$$\Sigma F_y = N - m_1 g \cos\theta = 0$$

$$\Sigma F_x = T + \mu_k N - m_1 g \sin\theta = -m_1 a$$

Block Two

$$\Sigma F_y = T - m_2 g = m_2 a$$

If the final value of "a" is positive, our assumption was correct.. If the value of "a" is negative, our assumption was incorrect, and we would expect the block to move up the plane.

TRUE OR FALSE? In this problem, the y-axis was taken perpendicular to the plane.

[b] CORRECT ANSWER: zero

According to the scale, the man is weightless. Although the Earth's mass still attracts the mass of the man, the following relationships develop:

1. The mass of the man accelerates downward at 32 ft/sec^2.
2. $\Sigma F_y = N - mg = -mg$
3. Thus, N must be zero.

[a] CORRECT ANSWER: B

B - Applying Newton's second law in the vertical direction to weight w and the elevator cage yields

Elevator Cage $\qquad\qquad$ w

$$T - 1600\ \text{lb} - f = \frac{1600\ \text{lb}}{g}\,a \quad (1) \qquad T - w = -\frac{w}{g}\,a \quad (2)$$

where f = 100 lb, the force of friction.

Note that the magnitudes of T and a are the same in both equations since the masses are connected by a string. Eliminating T and solving the simultaneous equations for w, we obtain

$$w = \frac{1600a + 1600g + fg}{g - a}$$

$$= \frac{1600\ \text{lb} \times 8\ \text{ft/sec}^2 + 1600\ \text{lb} \times 32\ \text{ft/sec}^2 + 100\ \text{lb} \times 32\ \text{ft/sec}^2}{32\ \text{ft/sec}^2 - 8\ \text{ft/sec}^2}$$

$$= 2800\ \text{lb}$$

TRUE OR FALSE? If the force of friction could be made smaller you would expect an acceleration greater than 8 ft/sec^2 when w = 2800 lb.

[b] CORRECT ANSWER: 185 lb

The sum of forces in the y-direction acting on the man yields

$$N - mg = ma$$

where N is the force of the scale on the person. Solving for N, we have

$$N = m(g + a)$$

$$= 185\ \text{lb}$$

The scale transmits this force to the man and thus his weight appears to increase to 185 lb.

TRUE OR FALSE? The normal force acting on the man is applied in a direction opposite that of his weight.